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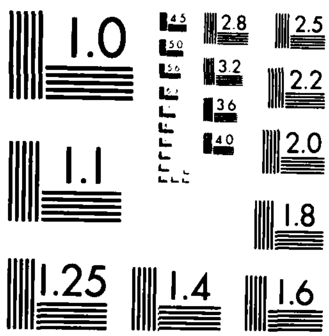

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NAVAL POSTGRADUATE SCHOOL
Monterey, California

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ON NONCOHERENT DIGITAL RECEIVERS

by

Hae Yeon Joo

December 1984

Thesis Advisor: Daniel Bukofzer

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The effects of various jamming waveforms on conventional binary incoherent digital receivers was analyzed in terms of resulting receiver performance (i.e., receiver probability of error).

Probability density functions associated with the test statistic generated by incoherent receivers under the influence of noise and jamming have obtained.

Due to the complexity of the mathematical expressions specifying receiver probability of error in closed form, no attempt has been made to obtain absolute optimum jamming waveforms operating against binary incoherent receiver. Therefore near optimum jammer signals were proposed, studied, and evaluated.

The effect of a varying threshold on receiver performance was investigated and a jamming strategy involving use of an FM jammer was considered, and its effect evaluated.

Graphical results are presented that highlight the mathematical results obtained.

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An Analysis Of Jamming Effects on Noncoherent Digital Receivers

by

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The effect of various jamming waveforms on conventional binary incoherent digital receivers was analyzed in terms of resulting receiver performance (i.e., receiver probability of error).

Probability density functions associated with the test statistic generated by incoherent receivers under the influence of noise and jamming have been obtained.

Due to the complexity of the mathematical expressions specifying receiver probability of error in closed form, no attempt has been made to obtain absolute optimum jamming waveforms operating against binary incoherent receivers. Therefore near optimum jammer signals were proposed, studied, and evaluated.

The effect of a varying threshold on receiver performance was investigated and a jamming strategy involving use of an FM jammer was considered, and its effect evaluated.

Graphical results are presented that highlight the mathematical results obtained.

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I. INTRODUCTION

A. COHERENT CORRELATION RECEIVER

The optimum receiver which will detect the presence of a signal or discriminate between two different signals in the presence of additive white Gaussian noise, is the well-known coherent correlator receiver structure which has two equivalent forms, as shown in Figure 6.1 [Ref. 2].

The binary communication problem is modeled using hypothesis testing theoretic principles in which one of two signals, $S_1(t)$ or $S_0(t)$, is received in the time interval $[0,T]$. Due to the presence of the noise, the observable signal r(t) takes on one of the two following forms

$$H_1: r(t) = S_1(t) + n(t) \qquad 0 \leq t \leq T$$

or

$$H_0: r(t) = S_0(t) + n(t) \qquad 0 \leq t \leq T$$

where the noise n(t) is assumed to be a sample function of a stationary white Gaussian process having zero mean and power spectral density (PSD) level $N_0/2$ watts/Hz.

The optimum receiver (in the sense of minimum probability error, Pe) generates the test statistic G, where G is given by

$$G = (r, S_d) + \frac{1}{2}\left[\|S_0\|^2 - \|S_1\|^2\right]$$

or equivalently

$$G = (r, S_1) - (r, S_0) + \frac{1}{2}\left[\|S_0\|^2 - \|S_1\|^2\right]$$

and compares it to a threshold in order to decide whether signal $S_1(t)$ or $S_0(t)$ has been transmitted in the time interval [0,T]. The inner product notation $(\cdot,\cdot)$ implies

$$(x,y) = \int_0^T x(t)y(t)\,dt$$

and the norm notation $||\cdot||$ indicates

$$\|x\|^2 = (x,x) = \int_0^T x^2(t)\,dt$$

In order to compute receiver probability of error, it is necessary to determine the probability density function of G conditioned on the hypotheses H_1 and H_0. Denoting these density functions $P_1(G)$ and $P_0(G)$ depending on whether $S_1(t)$ or $S_0(t)$ respectively was transmitted, the error probability P_e can be expressed as

$$P_e = P(H_1)\int_{-\infty}^{\gamma} P_1(G)\,dG + P(H_0)\int_{\gamma}^{\infty} P_0(G)\,dG$$

where

$$\gamma = \frac{N_0}{2}\,\ln\frac{P(H_0)}{P(H_1)}$$

and $P(H_1)$ denotes the priori probability that the hypothesis H_1 is true. It can readily be shown that if $P(H_1) = 1/2$,

$$P_e = \int_{\left[(1-\rho)E/N_0\right]}^{\infty} \frac{1}{\sqrt{2\pi}}\, e^{-z^2/2}\,dz \qquad (1.1)$$

where

$$E \triangleq \frac{1}{2}\int_0^T \left[S_1^2(t) + S_0^2(t)\right]\,dt$$

and

$$\rho \triangleq \frac{1}{E}\int_0^T S_1(t)S_0(t)\,dt$$

We can interpret E as the average energy per bit and ρ as the normalized signal cross correlation. Using the following definition of the error function

$$\mathrm{erfc}_{*}(x) = \int_{x}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz$$

Equation 1.1 can be written as

$$P_e = \mathrm{erfc}_{*}\left[\sqrt{(1-\rho)E/N_0}\right]$$

From the above equation it can be noted that as $(1-\rho)E/N_0$ increases, the error probability decreases. For fixed E/N_0 therefore, the optimum signals choice is one for which ρ = -1. This occurs whenever $S_1(t) = -S_0(t)$. A system employing this choice of signal is known as an optimum antipodal signaling binary communication system.

B. JAMMING OF COHERENT RECEIVERS

1. General

The objective of this section is to summarize previously derived results on the performance of the coherent binary (optimum) receiver of Figure 6.1 operating in a jamming ...ment. That is, the transmitted signal is interfered by the presence of a jamming waveform as well as additive white Gaussian noise [Ref. 1]. The signal appearing at the front-end of the receiver can be mathematically modeled by

$$H_1 : \; r(t) = S_1(t) + n(t) + n_j(t) \qquad 0 \le t \le T \tag{1.2}$$
or
$$H_0 : \; r(t) = S_0(t) + n(t) + n_j(t) \qquad 0 \le t \le T$$

where $n_j(t)$ is a jammer waveform modeled as deterministic, yet unknown to the receiver.

The resulting average probability of error is given by

$$\begin{aligned} P_e &= P_F \cdot P(H_0) + P_M\, P(H_1) \\ &= P(H_0) \int_{\eta^2}^{\infty} f_{G^2}(g|H_0)\, dg + P(H_1) \int_{-\infty}^{\eta^2} f_{G^2}(g|H_1)\, dg \end{aligned}$$

where η is the threshold with which G is compared in order to decide which is the true hypothesis. Observe that

$$f_{G^2}(g|H_1) = \quad f_{G^2|\theta}(g|H_1, \theta)\, f_\theta(\theta)\, d\theta$$

The first integral in the above expression for Pe is P_F (probability of false alarm). It can be expressed as follows

$$\begin{aligned} &\int_{\eta^2}^{\infty} \frac{1}{\sigma^2} \exp\left(-\frac{g + S'}{2\sigma^2}\right) I_0\left(\frac{\sqrt{gS'}}{\sigma^2}\right) dg \\ &= \int_{\eta/\sigma}^{\infty} V \exp\left(-\frac{V^2 + \alpha'^2}{2}\right) I_0(\alpha' V)\, dV \\ &= Q(\alpha', \eta/\sigma) \end{aligned}$$

where

$$Q(\alpha, \beta) = \int_{\beta}^{\infty} V \exp\left(-\frac{V^2 + \alpha^2}{2}\right) I_0(\alpha V)\, dV$$

is a well-known tabulated function called the Marcum Q-function. The second integral in the above expression for Pe is P_M (probability of miss). It can be expressed as follows

$$\int_0^{\eta^2} \left[\int_{\infty}^{\infty} f_{G^2|\theta}(g|H_1, \theta)\, f_\theta(\theta)\, d\theta \right] dg \qquad (2.3)$$

$$f_{G^2|\theta}(g|H_1,\Theta) = \frac{1}{2\sigma^2}\exp\left(-\frac{g+S}{2\sigma^2}\right) I_0\left(\frac{\sqrt{gS}}{\sigma^2}\right) \quad , \quad g \geq 0$$

and zero otherwise. The variance σ^2 is defined above and becomes

$$\begin{aligned} S &= m^2_{X|\theta} + m^2_{Y|\theta} = E^2\left\{X|H_1,\Theta\right\} + E^2\left\{Y|H_1,\Theta\right\} \\ &= \left[(AS_\theta,S) + (n_j,S)\right]^2 + \left[(AS_\theta,C) + (n_j,C)\right]^2 \end{aligned} \tag{2.1}$$

Due to our assumption $W_cT = n\pi$, we obtain

$$\begin{aligned} S = \left(\frac{AT}{2}\right)^2 &+ AT\left[(n_j,S)\cos\Theta + (n_j,C)\sin\Theta\right] \\ &+ (n_j,S)^2 + (n_j,C)^2 \end{aligned}$$

Similarly, if r(t) consists of noise and jammer only, then X and Y are also independent Gaussian random variables with

$$E\left\{X|H_0\right\} = (n_j,S) \triangleq m_X$$

and

$$E\left\{Y|H_0\right\} = (n_j,C) \triangleq m_Y$$

The density function of G^2 assuming no signal is sent is

$$f_{G^2}(g|H_0) = \frac{1}{2\sigma^2}\exp\left(-\frac{g+S'}{2\sigma^2}\right) I_0\left(\frac{\sqrt{gS'}}{\sigma^2}\right) \quad , \quad g \geq 0$$

and zero otherwise, where

$$S' = m_X^2 + m_Y^2 = (n_j,S)^2 + (n_j,C)^2 \tag{2.2}$$

where S_θ is used in place of $\sin(W_c t + \theta)$. It can also be seen that

$$\mathrm{Var}\{X|H_1,\theta\} = E\{(n,S)^2\}$$

$$= \frac{N_0 T}{4}\left[1 - \frac{\sin 2W_c T}{2W_c T}\right]$$

$$= \mathrm{Var}\{X|H_0\}$$

and similarly

$$\mathrm{Var}\{Y|H_1,\theta\} = \frac{N_0 T}{4}\left[1 + \frac{\sin 2W_c T}{2W_c T}\right]$$

$$= \mathrm{Var}\{Y|H_0\}$$

For convenience we assume $W_cT = n\pi$, where n is an integer. Thus the sinc terms above vanish resulting in $\mathrm{var}(X\ |H_1, \theta)$ $= \mathrm{var}(Y\ |H_1, \theta) = N_0T/4 \triangleq \sigma^2$. If this assumption is not made, an additional term results. However, if $\pi/W_c \ll T$, the additional sinc term is small and may be neglected. Furthermore the covariance

$$E\left\{\left[X - E\{X|H_i,\theta\}\right]\cdot\left[Y - E\{Y|H_i,\theta\}\right]\Big|\, H_i,\theta\right\}$$
$$= E\{(n,S)\,(n,C)\}$$
$$= \frac{N_0 T}{4}\left[\frac{1 - \cos 2W_c T}{2W_c T}\right] = 0 \qquad i = 0,1$$

provided the assumptions on W_c hold. This implies that for any given value of θ, both X and Y are uncorrelated Gaussian random variables and therefore statistically independent. The density function of G^2 conditioned on the phase θ is noncentral Chi-Squared distributed, and is given by

$$H_0 : \; r(t) = n(t) + n_j(t) \qquad 0 \leq t \leq T$$

where $n_j(t)$ is the jammer waveform present during the time interval $[0,T]$.

In the absence of $n_j(t)$, the optimum receiver for the binary ASK problem is well-known. Its derivation is well documented in the statistical detection theory literature [Ref. 2]. The receiver structure is shown in Figure 6.2

In this section, the effect of $n_j(t)$ on this receiver is analyzed by evaluating the resulting Pe, under the assumption that $n_j(t)$ is a deterministic jammer waveform, however unknown to the receiver itself. Receiver performance requires determination of the statistics of either G^2 or G, where G^2 is the output of the quadrature detector and is given by

$$G^2 = X^2 + Y^2$$

where

$$X = \int_0^T r(t) \sin W_c t \, dt \triangleq (r,S)$$
$$Y = \int_0^T r(t) \cos W_c t \, dt \triangleq (r,C)$$

Provided that the random variable Θ is fixed to some value θ, X and Y are conditional Gaussian random variables with

$$E\left\{X|H_1,\Theta\right\} = (AS_\theta,S) + (n_j,S) \triangleq m_{x|\theta}$$

and

$$E\left\{Y|H_1,\Theta\right\} = (AS_\theta,C) + (n_j,C) \triangleq m_{y|\theta}$$

II. ANALYSIS OF JAMMING ON ASK

A. GENERAL

In this chapter, the effect of a deterministic jammer waveform on the performance of an incoherent ASK receiver will be investigated.

The signal at the front-end of the receiver is given by Equation 1.8 with the modification that under either hypothesis, a jamming waveform $n_j(t)$ is present during the time interval [0,T].

For the coherent ASK receiver, the optimum energy constrained jammer waveform could be obtained by using Equation 1.5 . However, the complexity of the mathematical expression for probability of error in the noncoherent case makes it very difficult if not impossible to derive the optimum energy constrained jammer waveform in closed form as will be seen in the sequel.

A reasonable postulation is that the optimum jamming waveform for the coherent ASK receiver can act as a good jammer for the incoherent ASK receiver also. Thus, such near optimum jammer signals are studied and evaluated in terms of their effect on the performance of the noncoherent ASK receiver.

B. ANALYSIS WITH NEAR OPTIMUM JAMMER

Analysis of the incoherent receiver starts from the mathematical model of the receiver front-end input waveform r(t) given by either

$$H_1 : \; r(t) = A \sin (W_c t + \theta) + n(t) + n_j(t) \qquad 0 \leq t \leq T$$

or

where we assume that the signal amplitudes even though random, remain constant over a T - second interval. The noise is modeled as in the previous analysis, and the signal amplitudes have (a priori) probability density functions given by

$$f_A(a) = \frac{a}{A_o^2} \exp\left(- \frac{a^2}{2 A_o^2}\right) \qquad a \geq 0$$

and

$$f_B(b) = \frac{b}{A_o^2} \exp\left(- \frac{b^2}{2 A_o^2}\right) \qquad b \geq 0$$

where $A_o^2 = E\{A^2\} = E\{B^2\}$.

The results associated with ASK incoherent receivers can be applied toward obtaining the optimum incoherent FSK receiver. This is worked out in Reference 2. The resulting receiver structure is shown in Figure 6.5 . Individual channel matched filters shown in Figure 6.5 can be essentially replaced with tuned bandpass filters having center frequencies corresponding to the "mark" and "space" frequencies W_1 and W_o.

an envelope detector and a sampler as shown in Figure 6.4 [Ref. 2].

3. FSK

For binary frequency shift keying (FSK), the received signals at the front end of the receiver in the presence of noise only can be mathematically modeled by either

$$H_1 : \; r(t) = A \sin (W_1 t + \Theta) + n(t) \qquad 0 \leq t \leq T$$

or

$$H_0 : \; r(t) = A \sin (W_0 t + \Phi) + n(t) \qquad 0 \leq t \leq T$$

where the additive noise is again assumed to be zero mean, white Gaussian, with power spectral density level $N_0/2$ watts/Hz. Usually the frequencies W_1 and W_0 differ substantially. The phases Θ and Φ are statistically independent random variables, uniformly distributed over the interval $[0,2\pi]$.

Insofar as the signal amplitudes is concerned, two cases are distinguished. In the first case, the information bearing signal amplitudes are known and equal. In the second case, the information bearing signal amplitudes are treated as independent random variables that are Rayleigh distributed due to multipath propagation. For this second case, the following hypothesis testing theoretic model is used

$$H_1 : \; r(t) = A \sin (W_1 t + \Theta) + n(t) \qquad 0 \leq t \leq T$$

and

$$H_0 : \; r(t) = B \sin (W_0 t + \Phi) + n(t) \qquad 0 \leq t \leq T$$

2. ASK (On - Off keying)

In the case of incoherent ASK (On - Off keying), the signals at the front-end of the receiver in the presence of noise only can be mathematicaly modeled by either

$$H_1 : \ r(t) = A \sin (W_c t + \Theta) + n(t) \qquad 0 \leq t \leq T \qquad (1.8)$$

or

$$H_0 : \ r(t) = n(t) \qquad 0 \leq t \leq T$$

The amplitude A, the frequency Wc and the time of arrival are assumed to be known except that the phase Θ is modeled as a random variable having an a priori density function

$$f_\Theta(\theta) = \begin{cases} \frac{1}{2\pi} & 0 \leq \theta \leq 2\pi \\ 0 & \text{otherwise} \end{cases}$$

The additive noise is again assumed to be zero mean, white Gaussian, with power spectral density level $N_0/2$ watts/Hz.

In the absence of a jamming waveform $n_j(t)$, the optimum receiver for decoding the binary information (which is transmitted via the use of either a sinusoid that is incoherently received, or no signal at all) in each interval of duration T sec, is the well-known quadrature receiver. Its structure is shown in Figure 6.2 [Ref. 2].

There is an alternate form of the quadrature receiver obtained by replacing the correlators with matched filters having an impulse response given by h(t) = sin Wc(T - t) and h(t) = cos Wc(T - t), 0 < t < T, and sampling the outputs at t = T (Figure 6.3) [Ref. 2].

Still another important alternate form of the quadrature receiver is an incoherent matched filter followed by

$$P_e = \frac{1}{2}\left[\int_{\sqrt{SNR}\,(\sqrt{\alpha} - \sqrt{2JSR})}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\,dx + \int_{-\infty}^{-\sqrt{SNR}\,(\sqrt{\alpha} + \sqrt{2JSR})} \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\,dx\right]$$

where now

$$\rho = \frac{2A_1A_0}{A_1^2 + A_0^2}$$

and

$$\alpha = 1 - \rho = \frac{(A_1 - A_0)^2}{A_1^2 + A_0^2}$$

The 'break point' for ASK occurs at JSR = $\alpha/2$. Since $\alpha < 1$, in terms of 'break point' efficiency, PSK is highest with a 'break point' occuring at a JSR of 1. FSK is next highest in efficiency with a 'break point' occuring at a JSR of 1/2, and ASK is lowest in efficiency with a 'break point' occuring at JSR < 1/2.

C. INCOHERENT RECEIVERS

1. General

In the coherent systems considered, the information bearing signals were assumed to be known exactly at the receiver.

In noncoherent systems, the phase of the carrier signals is not available at the receiver so that the phase is treated as a random variable uniformly distributed over $[0, 2\pi]$. As such, we may expect the performance of an incoherent system to be degraded in comparison to the performance of the corresponding coherent system. However, because of their simplicity, incoherent systems are widely used in many applications.

with the constraint that $(W_1 - W_0)T = l\pi$; $(W_1 + W_0)T = m\pi$; l and m integers. Finally for ASK modulation

$$S_1(t) = A_1 S(t) \quad ; \quad S_0(t) = A_0 S(t) \qquad 0 \le t \le T$$

where we assume that $||S|| < \infty$ and for convenience, that $A_1 > A_0$. The optimum jammer waveforms and resulting receiver performances can be obtained using Equations 1.5 and 1.7. Thus the optimum jammer for PSK is given by

$$n_j(t) = \sqrt{P_{n_j}}\sqrt{\frac{2}{T}}\cos W_c t \qquad 0 \le t \le T$$

Since $\rho = -1$ for PSK, we have

$$P_e = \frac{1}{2}\left[\int_{\sqrt{2SNR}\,(1-\sqrt{JSR})}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-x^2/2}\,dx + \int_{-\infty}^{-\sqrt{2SNR}\,(1+\sqrt{JSR})}\frac{1}{\sqrt{2\pi}}e^{-x^2/2}\,dx\right]$$

For FSK modulation, the optimum jammer is given

$$n_j(t) = \sqrt{P_{n_j}}\,\frac{2}{\sqrt{T}}\sin\frac{1}{2}(W_1 - W_0)t\cos\frac{1}{2}(W_1 + W_0)t$$

Using the assumptions on W_1 and W_0, the value of ρ is zero. Thus Equation 1.7 becomes

$$P_e = \frac{1}{2}\left[\int_{\sqrt{SNR}\,(1-\sqrt{2JSR})}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-x^2/2}\,dx + \int_{-\infty}^{-\sqrt{SNR}\,(1+\sqrt{2JSR})}\frac{1}{\sqrt{2\pi}}e^{-x^2/2}\,dx\right]$$

Finally, for ASK modulation, the optimum jammer is given by

$$n_j(t) = \sqrt{P_{n_j}}\,\frac{S(t)}{\|S\|}$$

and the probability of error becomes

$\frac{P_{n_j}}{E} \triangleq$ JSR : jammer to signal ratio

and observing that

$$\|S_d\|^2 = \int_0^T \left[S_1(t) - S_0(t)\right]^2 dt = 2\,E\,(1-\rho)$$

Then, the probability of error (Equation 1.6) becomes

$$P_e = \frac{1}{2}\left[\int_{\sqrt{SNR}\left(\sqrt{1-\rho}-\sqrt{2JSR}\right)}^{\infty} \frac{1}{\sqrt{2\pi}}\, e^{-x^2/2}\, dx + \int_{-\infty}^{-\sqrt{SNR}\left(\sqrt{1-\rho}+\sqrt{2JSR}\right)} \frac{1}{\sqrt{2\pi}}\, e^{-x^2/2}\, dx\right] \quad (1.7)$$

Analysis of Equation 1.7 indicates the fact that for a value of JSR beyond a so-called 'break point', that is JSR > $(1-\rho)/2$, an increasing SNR causes Pe to become worse, i.e., it increases.

3. Optimum Jammer Waveforms for PSK, FSK and ASK

The effectiveness of a deterministic jammer waveform on various modulation techniques will now be presented for PSK modulation, where

$$S_1(t) = A \sin W_c t \quad ; \quad S_0(t) = A \sin(W_c t + \pi) \qquad 0 \le t \le T$$

with the constraint that $W_cT = n\pi$; n an integer. Also, results for FSK modulation will be shown, where

$$S_1(t) = A \sin W_1 t \quad ; \quad S_0(t) = A \sin W_0 t \qquad 0 \le t \le T$$

$$|d| = |(n_j, S_d)| \le \|n_j\| \cdot \|S_d\| \tag{1.4}$$

with equality if $n_j(t)$ is proportional to $S_d(t)$. Defining $||n_j|| = \sqrt{P_{n_j}}$, where P_{n_j} is the jammer energy, the term $|d| \to \infty$ implies that $P_{n_j} \to \infty$ when $||S_d|| < \infty$. Since it is not possible to have infinite jammer energy, one must constrain $n_j(t)$ such that P_{n_j} is finite. From the Cauchy-Schwarz inequality, Equation 1.4 can be made into an equality if

$$n_j(t) = k \cdot S_d(t)$$

where K is a constant of proportionality. If $||n_j|| = \sqrt{P_{n_j}}$, K must be set to the value $\sqrt{P_{n_j}}/||S_d||$. Thus $|d|$ is maximized by setting

$$n_j(t) = \sqrt{P_{n_j}}\; S_d(t)/\|S_d\| \tag{1.5}$$

and from the above discussion, this results in Pe being maximized also. Substituting Equation 1.5 into the probability of error expression (Equation 1.3) yields

$$P_e = \frac{1}{2}\left[\int_{\sqrt{\frac{2}{N_0}}\left[\frac{\|S_d\|}{2} - \sqrt{P_{n_j}}\right]}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\,dx + \int_{-\infty}^{-\sqrt{\frac{2}{N_0}}\left[\frac{\|S_d\|}{2} + \sqrt{P_{n_j}}\right]} \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\,dx\right] \tag{1.6}$$

It is possible to put Equation 1.6 in a more meaningful form by defining

$$\frac{E}{N_0} \triangleq SNR \quad : \text{ signal to noise ratio}$$

where

$$S_{\gamma} \triangleq \left[\frac{2}{N_o \, \|S\|^2} \right]^{1/2} ;$$

$$N_J \triangleq \frac{\|S_d\|^2}{2} ; \quad d \triangleq (n_j, S_J)$$

From the jammer standpoint, the optimum jammer waveform must be chosen in such a way that it maximizes the receiver probability of error. A determination of the optimum jammer waveform can be made by first evaluating the derivative of Pe with respect to d, which is the cross correlation between the jammer waveform and the signal difference $S_d(t)$. Based on Equation 1.3, we can show that

$$\frac{\partial P_e}{\partial d} = \frac{S_{\gamma}}{\sqrt{2\pi}} e^{-S_{\gamma}^2 \left[N_J^2 + d^2 \right] / 2} \sinh S_{\gamma}^2 N_J \cdot d$$

Due to the behavior of the hyperbolic sine function, we observe that

$$\frac{\partial P_e}{\partial d} = \begin{cases} > 0 & , d > 0 \\ = 0 & , d = 0 \\ < 0 & , d < 0 \end{cases}$$

and

$$\frac{\partial^2 P_e}{\partial d^2} = \frac{S_{\gamma}^3 \, N_J}{\sqrt{2\pi}} \exp \left[- S_{\gamma}^2 (d + N_J)^2 / 2 \right] > 0$$

for all values of d. Since Pe is a monotonic function of d, it is apparent that making d as large as possible in magnitude results in the largest possible increase in Pe. In the limit as $|d| \to \infty$, it is easily seen that Pe --> 1/2. However, from the Cauchy - Schwarz inequality

2. Jammer Optimization

The effect of a jammer waveform on the optimum coherent receiver is now presented in terms of its impact on the receiver probability of error.

In order to determine the performance of the receiver, we analyze the test statistic G, which is given by

$$G = (r, S_J) + \frac{1}{2}\left[\|S_0\|^2 - \|S_1\|^2\right]$$

with r(t) now given by Equation 1.2 . Due to the assumption of a Gaussian zero mean noise having PSD level $N_o/2$ watts/Hz, it is simple to show that G is a conditionally Gaussian random variable with mean value

$$E\left\{G|S_i\right\} = (S_i, S_J) + (n_j, S_J) + \frac{1}{2}\left[\|S_0\|^2 - \|S_1\|^2\right] \quad , i = 0,1$$

and variance

$$\mathrm{Var}\left\{G|S_i\right\} = \frac{N_o}{2}\|S_J\|^2 \qquad i = 0,1$$

With the mean and variance known, the resulting conditional density function of G is given by

$$f_G(g|S_i) = \frac{1}{\sqrt{2\pi\cdot\frac{N_o}{2}\|S_J\|^2}}\cdot$$

$$\exp\left\{-\frac{\left[g - (S_i, S_J) - (n_j, S_J) - \frac{1}{2}\left[\|S_0\|^2 - \|S_1\|^2\right]\right]^{1/2}}{2\cdot\frac{N_o}{2}\cdot\|S_J\|^2}\right\} \quad i = 0,1$$

From the general probability of error expression (Equation 1.1) and an approriate change of variables in the integrals, when $P(H_1) = P(H_o) = 1/2$ the expression for Pe becomes

$$P_e = \frac{1}{2}\left[\int_{S_\vartheta[N_J - d]}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-x^2/2}dx + \int_{-\infty}^{-S_\vartheta[N_J + d]}\frac{1}{\sqrt{2\pi}}e^{-x^2/2}dx\right] \qquad (1.3)$$

$$= \int_{-\infty}^{\infty} f_{\theta}(\theta) \left[\int_{0}^{\eta^2} f_{G^2|\theta}(g|H_1, \theta)\, dg \right] d\theta$$

where in the second equality the order of the integration has been changed. The inner integral of Equation 2.3 can be expressed in term of the Marcum Q-function as

$$\int_0^{\eta^2} \frac{1}{2\sigma^2} \exp\left(-\frac{g+S}{2\sigma^2}\right) I_0\left(\frac{\sqrt{gS}}{\sigma^2}\right) dg$$
$$= \int_0^{\eta/\sigma} V \exp\left(-\frac{V^2+\alpha^2}{2}\right) I_0(\alpha V)\, dV$$
$$= 1 - Q(\alpha, \eta/\sigma)$$

where $\alpha = \sqrt{S}/\sigma$. Therefore the probability of error can be written as

$$P_e = P(H_0)\, Q\left(\frac{\sqrt{S'}}{\sigma}, \eta/\sigma\right) + P(H_1) \left[1 - \frac{1}{2\pi} \int_0^{2\pi} Q\left(\frac{\sqrt{S}}{\sigma}, \eta/\sigma\right) d\theta \right] \tag{2.4}$$

where the dependence on θ is imbedded in the term $\sqrt{S}$ and the threshold η for the jammer absent case is obtained as the solution to the equation

$$e^{-A^2T/2N_0} I_0(2A\eta/N_0) = R \quad ; \quad R = \frac{P(H_0)}{P(H_1)}$$

which can be equivalently put in the form

$$e^{-A^2T/2N_0} I_0\left[\sqrt{A^2T/N_0}\,(\eta/\sigma)\right] = R$$

The $I_0(\cdot)$ function used here and also previously used in conjunction with the development of Pe is the modified

Bessel function of the first kind. The appropriate setting of the threshold is an important issue for both coherent as well as noncoherent receivers. One could use the threshold setting that would be derived from the analysis of receivers operating in additive white Gaussian noise interference only. This approach can be quite unsatisfactory as demonstrated in Reference 1. A better approach would be to obtain Pe as a function of the threshold, and then search for the threshold that minimizes Pe. While this approach is intuitively appealing, it is often mathematically intractable. While the threshold issue is not addressed in this particular section, it will be discussed in more detail in the sequel and simulation results will be presented.

If the definition of average signal energy previously introduced is used, we have $E = A^2T/4$ which is reduced in half in comparison to the binary signal transmission case, due to the fact that the information bearing signal(s) do not have equal energy. In order to afford comparisons with the coherent receiver case, we will implicitely boost the value of signal amplitude A, to obtain $E = A^2T/2$ in order to have agreement with previous cases insofar as signal energies is concerned. Thus the threshold determination equation now becomes

$$e^{-SNR} I_0 \left[\sqrt{2SNR} \, (\eta / \sigma) \right] = R \quad ; \quad R = \frac{P(H_0)}{P(H_1)} \tag{2.5}$$

If we assume $P(H_1) = P(H_0) = 1/2$, then

$$P_e = \frac{1}{2} \left[1 - \frac{1}{2\pi} \int_0^{2\pi} Q\left(\frac{\sqrt{S}}{\sigma}, \eta / \sigma \right) d\theta + Q\left(\frac{\sqrt{S'}}{\sigma}, \eta / \sigma \right) \right]$$

and the threshold setting equation becomes

$$e^{-SNR} I_0 \left[\sqrt{2SNR} \, (\eta / \sigma) \right] = 1$$

If we are to find the optimum jammer waveform so as to maximize Pe, an attempt must be made to solve

$$\frac{\partial P}{\partial S} = 0 \quad \text{and} \quad \frac{\partial P}{\partial S'} = 0$$

Unfortunately the resultant equations are mathematically involved and do not appear readily solvable for $n_j(t)$. It seems however that a 'good' jammer waveform can be postulated based on the results obtained for coherent ASK. It was found for that case that the optimum $n_j(t)$ is a tone at the carrier frequency. Thus the following jammer waveform can be used as a potential near optimum jammer, namely

$$n_j(t) = \sqrt{P_{n_j}}\sqrt{\frac{2}{T}} \ \sin W_c t \ , \ 0 \leq t \leq T \tag{2.6}$$

Observe that with this choice, $||n_j||^2 = P_{n_j}$. The probability of error Pe can now be determined using the threshold setting equation (Eqn. 2.5) and the previously derived expressions for S and S'. The effect of the near optimum jammer waveform on the receiver (i.e., incoherent receiver performance) can be analyzed by evaluating Pe as a function of $n_j(t)$ using Equation 2.6 . Note that in Equation 2.6 the jammer energy is P_{n_j}. It can be shown that

$$(n_j,S) = \int_0^T \sqrt{P_{n_j}}\sqrt{\frac{2}{T}} \ \sin^2 W_c t = \sqrt{\frac{P_{n_j} T}{2}}$$

$$(n_j,C) = \int_0^T \sqrt{P_{n_j}}\sqrt{\frac{2}{T}} \ \sin W_c t \cos W_c t \ dt = 0$$

Then from Equations 2.1 and 2.2, the probability of error Pe given by Equation 2.4 can be expressed in terms of SNR (signal-to-noise ratio) and JSR (jammer-to-signal ratio) using the fact that

$$\frac{S}{\sigma^2} = 2\ \mathrm{SNR}\ (1 + 2\sqrt{\mathrm{JSR}}\cos\theta + \mathrm{JSR})$$
$$\frac{S'}{\sigma^2} = 2\ \mathrm{SNR}\cdot\mathrm{JSR}$$

and the fact that η/σ can be obtained from Equation 2.5.

For equally likely hypotheses, the probability of receiver error (Equation 2.4) can be expressed by

$$P_e = \frac{1}{2}\left[1 + Q\left(\sqrt{2\mathrm{SNR}\ \mathrm{JSR}}\ ,\ \eta/\sigma\right) - \frac{1}{2\pi}\int_0^{2\pi} Q\left(\sqrt{2\mathrm{SNR}\ (1 + 2\sqrt{\mathrm{JSR}}\cos\theta + \mathrm{JSR})}\ ,\ \eta/\sigma\right) d\theta\right] \tag{2.7}$$

From Equation 2.7, Pe greater than 0.5 can occur if

$$Q(\alpha, \beta) > \frac{1}{2\pi} Q(\alpha', \beta)\ d\theta \tag{2.8}$$

where

$$\alpha' = \sqrt{2\mathrm{SNR}\ (1 + 2\sqrt{\mathrm{JSR}}\cos\theta + \mathrm{JSR})}$$
$$\alpha = \sqrt{2\mathrm{SNR}\ \mathrm{JSR}} \quad \text{and}\ \beta = \eta/\sigma$$

Since the periodic sinusoidal function imbedded in α' is integrated for one period, the condition $\alpha \geq \alpha'$ might cause the condition of Equation 2.8 to be satisfied for a fixed value of β. Thus there is a possibility of obtaining Pe greater than 0.5 for a JSR value beyond the critical value of 0.25 which can be obtained from the condition $\alpha = \alpha'$, that is,

$$\sqrt{2\mathrm{SNR}\cdot\mathrm{JSR}} = \sqrt{2\mathrm{SNR}\ (1 + 2\sqrt{\mathrm{JSR}}\cos\theta + \mathrm{JSR})} \tag{2.9}$$
$$\sqrt{\mathrm{JSR}}\cos\theta = -\frac{1}{2}$$

Receiver performance in the presence of a jammer is expressed in terms of signal-to-noise ratio (SNR) and jammer-to-siganl ratio (JSR). A rectangular pulse of duration T seconds has amplitude spectrum AT sinc Tf, and B = 2/T is a rough measure of its bandwidth. Thus the expression of the form

$$\frac{A^2T/2}{N_o} = \frac{A^2/2}{\frac{N_o}{2}\cdot\frac{2}{T}} = \frac{A^2/2}{PSD\cdot B} \qquad (2.10)$$

can be interpreted as the ratio of signal power to noise power (SNR) in the signal bandwidth [Ref. 5]. The term JSR can be expressed by

$$\frac{P_{n_j}}{A^2T/2} = \frac{\text{jammer energy}}{\text{signal energy}} \qquad (2.11)$$

where P_{n_j} represents the jammer energy defined before.

C. VARIABLE THRESHOLDING EFFECT

It is apparent that when a fixed threshold value is used by the receiver, the effect of the jammer waveform having energy such that the resulting JSR value is above a certain level, is such that the receiver may be rendered inoperable.

It could therefore be suggested that setting the threshold based on the value $P(H_1)$ or $P(H_0)$ only (see Eqn. 2.5), may not be desireable. Values of threshold other than some fixed value may result in improved receiver performance, in other words, reduce the jamming effect.

Recall that the threshold setting equation is of the form

$$e^{-SNR} I_o(\sqrt{2SNR}\,\alpha) = R \qquad (2.12)$$

where $\alpha = \eta/\sigma$. we now can attempt to reduce Pe by an appropriate choice of threshold α. By varying the value of R (instead of using R = 1) we can obtain this threshold which denote α' from the expression

$$\alpha' = \frac{1}{\sqrt{2SNR}} I_0^{-1}(R\, e^{SNR})$$

where $I_0^{-1}(X)$ is the inverse modified Bessel function of the first kind. Using an approximation to $I_0(X)$ which is given by

$$I_0(x) \cong \frac{e^x}{\sqrt{2\pi x}} \quad \text{for } x \gg 1 ,$$

Equation 2.12 can be rewritten as

$$\begin{aligned} 4\sqrt{2SNR}\,\alpha' - 2\ln\alpha' \\ = 2\ln 2\pi + \ln(2SNR) + 4SNR + \ln R \end{aligned} \tag{2.13}$$

From Equation 2.13 , it can be recognized that when SNR is large, variation of the value of R does not significantly affect the threshold value α' because taking the logarithm of R reduces the effect of R further. Thus the large value of SNR suppresses the effect of variation of the term $\ln R$ on α'. This limited variable thresholding effect on the receiver which is under significant jamming is analyzed in graphical form in Chapter 5 for various values of R.

III. ANALYSIS OF JAMMING ON FSK

A. GENERAL

For binary incoherent FSK with a jammer present, the received signals under the two hypotheses are either

$$H_1 \ : \ r(t) = A \sin (W_1 t + \Theta) + n(t) + n_j(t) \qquad 0 \leq t \leq T$$

or

$$H_0 \ : \ r(t) = A \sin (W_0 t + \Phi) + n(t) + n_j(t) \qquad 0 \leq t \leq T$$

By separating the frequencies W_1 and W_0 sufficiently, we can form signals that are orthogonal, have equal energy, and have the same advantage of ease of generation.

The modified FSK receiver structure which is capable of varying the output of the each envelope detector (which is followed by a multiplier) is diagramed in Figure 6.6 . The optimum receiver for the case where no jammer is present can be derived from statistical decision theory and is shown in Figure 6.5 . This receiver can be obtained by setting α = 1/2 in the modified FSK receiver shown in Figure 6.6 .

In practice, incoherent FSK is widely used because of its simple receiver structure, its small performance penalty due to lack of phase coherence and its more efficient use of signal energies in comparison to incoherent ASK. In addition, we are not faced with the difficulty imposed by a threshold that must change with SNR as is the case with incoherent ASK. Thus, a receiver which is known to be optimum for incoherent FSK transmission, has been modified by including some channel weighting. This has been done in order to be able to determine whether or not such channel weighting can reduce the effect of the jammer.

This chapter is devoted to investigating the performance of the modified incoherent FSK receiver in the presence of jamming and additive white Gaussian noise. By letting α = 1/2 we can as a byproduct obtain the performance of the conventional incoherent FSK receiver of Figure 6.5 in the presence of jamming and additive white Gaussian noise.

B. ANALYSIS WITH NEAR OPTIMUM JAMMER

The modified receiver performance can be obtained by introducing a null hypothesis (no signal) as a dummy hypothesis and by following the same reasoning as in the analysis of incoherent ASK presented in the previous chapter.

The receiver function is to compare the envelopes at the output of each channel once every T seconds and decide in favor of the larger of the two envelopes (Figure 6.6). For the purpose of analysis, let us first assume that a 'mark' signal has been transmitted, that is, the hypothesis H_1 is assumed to be true. An error is committed if V_0 exceeds V_1. An error is also committed if V_1 is larger than V_0 when a 'space' signal has been transmitted, that is, the hypothesis H_0 is assumed to be true [Ref. 3].

Let Pe_1 denote the probability of the first type of error described above, which is expressed as Prob.($V_0 > V_1$ | H_1). Under the assumption that a 'mark' signal has been sent, the output q_1 of one of the envelope detectors is given by

$$q_1^2 = X_1^2 + Y_1^2 \quad .$$

where

$$X_1 = \int_0^T r(t) \sin W_1 t \, dt \quad ; \quad Y_1 = \int_0^T r(t) \cos W_1 t \, dt$$

Observe that X_1 and Y_1 conditioned on the phase and either of the two hypotheses are Gaussian random variables with

$$E\left\{X_1 \middle| H_1,\Theta\right\} = \int_0^T A \sin(W_1 t + \Theta) \sin W_1 t \, dt + \int_0^T n_j(t) \sin W_1 t \, dt \triangleq (S_1, S)_1 + (n_j, S)_1$$

and

$$E\left\{Y_1 \middle| H_1,\Theta\right\} = \int_0^T A \sin(W_1 t + \Theta) \cos W_1 t \, dt + \int_0^T n_j(t) \cos W_1 t \, dt \triangleq (S_1, C)_1 + (n_j, C)_1$$

where S_1 represents the function $A \sin(W_1 t + \theta)$, S represents the function $\sin W_1 t$ and C represents the function $\cos W_1 t$. Likewise, assuming again that $W_1 T = n\pi$, n an integer, and that $n(t)$ is zero mean white Gaussian noise with PSD level $N_0/2$ watts/Hz, we obtain

$$\mathrm{Var}\left\{X_1 \middle| H_1,\Theta\right\} = \mathrm{Var}\left\{Y_1 \middle| H_1,\Theta\right\} = \frac{N_0 T}{4}$$

Furthermore, it can be shown that

$$E\left\{\left[X_1 - E\left\{X_1 \middle| H_1,\Theta\right\}\right]\left[Y_1 - E\left\{Y_1 \middle| H_1,\Theta\right\}\right] \middle| H_1,\Theta\right\} = 0$$

so that the conditional r.v.'s X_1 and Y_1 are uncorrelated and therefore independent. The sum involving random variables X_1 and Y_1, and producing q_1^2, will result in a non-central Chi-Squared distribution so that

$$f_{Q_1^2|\theta}(q_1|H_1,\Theta) = \frac{1}{2\sigma^2} \exp\left(-\frac{q_1 + S_{11}}{2\sigma^2}\right) I_0\left(\frac{\sqrt{q_1 S_{11}}}{\sigma^2}\right) \qquad q_1 \geq 0$$

where

$$S_{11} = E^2\left\{X_1 \middle| H_1,\Theta\right\} + E^2\left\{Y_1 \middle| H_1,\Theta\right\}$$
$$= \left[(S_1, S)_1 + (n_j, S)_1\right]^2 + \left[(S_1, C)_1 + (n_j, C)_1\right]^2$$

and $\sigma^2 = N_0T/4$. Using standard random variable transformation techniques, it is not difficult to show that

$$f_{Q_1|\theta}(q_1|H_1,\theta) = \frac{q_1}{\sigma^2}\exp\left(-\frac{q_1^2+S_{11}}{2\sigma^2}\right) I_0\left(\frac{q_1\sqrt{S_{11}}}{\sigma^2}\right) \qquad q_1 \geq 0$$

so that

$$\begin{aligned} f_{Q_1}(q_1|H_1) &= f_{Q_1|\theta}(q_1|H_1,\theta)\, f_{\Theta}(\theta)\, d\theta \\ &= \frac{1}{2\pi}\int_0^{2\pi}\frac{q_1}{\sigma^2}\exp\left(-\frac{q_1^2+S_{11}}{2\sigma^2}\right) I_0\left(\frac{q_1\sqrt{S_{11}}}{\sigma^2}\right) d\theta \end{aligned} \tag{3.1}$$

where the dependence on θ is imbedded in the term S_{11}. We now need to obtain the statistics of the output of the multiplier following the upper envelope detector. (See Figure 6.6). That is, the probability density function $f_{V_1}(V_1 | H_1)$ needs to be derived. From standard transformation theory, using the relation $V_1 = 2(1-\alpha)q_1$, it can be shown that

$$\begin{aligned} f_{V_1}(V_1|H_1) &= \frac{1}{2(1-\alpha)} f_{Q_1}\left(\frac{V_1}{2(1-\alpha)}\bigg| H_1\right) \\ &= \frac{1}{4\pi(1-\alpha)}\int_0^{2\pi}\frac{\frac{V_1}{2(1-\alpha)}}{\sigma^2}\exp\left(-\frac{\left(\frac{V_1}{2(1-\alpha)}\right)^2+S_{11}}{2\sigma^2}\right)\cdot \\ &\qquad I_0\left(\frac{\frac{V_1}{2(1-\alpha)}\sqrt{S_{11}}}{\sigma^2}\right) d\theta \end{aligned} \tag{3.2}$$

On the other hand, the output of the lower envelope detector when H_1 is assumed to be the true hypothesis, is given by

$$q_o^2 = X_o^2 + Y_o^2$$

where

$$X_o = \int_0^T r(t) \sin W_o t \, dt \quad ; \quad Y_o = \int_0^T r(t) \cos W_o t \, dt$$

Following a similiar procedure as used above, it can be shown that

$$E\left\{Y_o | H_1, \theta\right\} = \int_0^T A \sin (W_1 t + \theta) \cos W_o t \, dt$$
$$+ \int_0^T n_j(t) \cos_o W t \, dt \triangleq (S_1, C)_o + (n_j, C)_o$$

and

$$E\left\{X_o | H_1, \theta\right\} = \int_0^T A \sin (W_1 t + \theta) \sin W_o t \, dt$$
$$+ \int_0^T n_j(t) \sin_o W t \, dt \triangleq (S_1, S)_o + (n_j, S)_o$$

Here, S and C have the same meaning previously defined except that the subscript "0" outside the inner products implies that we should interpret S as sin $W_o t$ and C as cos $W_o t$. It can also be demonstrated that

$$\mathrm{Var}\left\{X_o | H_1, \Theta\right\} = \mathrm{Var}\left\{Y_o | H_1, \Theta\right\} = \frac{N_o T}{4}$$

and also that

$$E\left\{\left[X_o - E\left\{X_o | H_1, \Theta\right\}\right]\left[Y_o - E\left\{Y_o | H_1, \Theta\right\}\right] \Big| H_1, \Theta\right\} = 0$$

so that the conditional r.v.'s X_o and Y_o are uncorrelated, hence independent. Thus similiar to Equation 3.1, the expression for the conditional density function of q_o becomes

$$f_{Q_o}(q_o|H_1) = \frac{1}{2\pi}\int_0^{2\pi} \frac{q_o}{\sigma^2}\exp\left(-\frac{q_o^2 + S_{01}}{2\sigma^2}\right)\cdot$$

$$I_0\left(\frac{q_o\sqrt{S_{01}}}{\sigma^2}\right) d\theta \quad , \quad q_o \geq 0$$

where

$$S_{01} = E^2\left\{X_o|H_1,\Theta\right\} + E^2\left\{Y_o|H_1,\Theta\right\}$$
$$= \left[(S_1,S)_o + (n_j,S)_o\right]^2 + \left[(S_1,C)_o + (n_j,C)_o\right]^2$$

and dependence on θ now is imbedded in the term S_{01}. Applying again the random variable transformation $V_o = 2\alpha q_o$, the conditional density function of V_o is given by

$$f_{V_o}(V_o|H_1) = \frac{1}{4\pi\alpha}\int_0^{2\pi} \frac{\frac{V_o}{2\alpha}}{\sigma^2} \exp\left(-\frac{\left(\frac{V_o}{2\alpha}\right)^2 + S_{01}}{2\sigma^2}\right)\cdot \qquad (3.3)$$

$$I_0\left(\frac{\frac{V_o}{2\alpha}\sqrt{S_{01}}}{\sigma^2}\right) d\theta$$

In order to compute the probability of error, we can now use the previous expressions for the conditional probability density functions which are derived assuming a 'mark' signal has been sent. That is, for a given value of V_1, an error is made if $V_o > V_1$. Thus the average error probability is found by averaging the conditional error probability given by

$$P(V_1) = \int_{V_1}^{\infty} f(V_o|H_1,V_1)\, dV_o$$

over all V_1. That is,

$$P_{e_1} = \text{Prob.}\left[V_o > V_1 | H_1\right] \qquad (3.4)$$

$$= \int_0^\infty \left[\int_{V_1}^\infty f_{V_0} \;(V_0 | H_1\;)\; dV_0 \right] f_{V_1} \;(V_1 | H_1\;)\; dV_1$$

Substituting for the density functions in Equation 3.4, from Equations 3.2 and 3.3, we obtain

$$P_{e_1} = \frac{1}{2\pi} \int_0^\infty \left[\int_0^{2\pi} \left\{ \int_{V_1}^\infty \frac{V_0}{\sigma^2} \exp\left(- \frac{\left(\frac{V_0}{2\alpha}\right)^2 + S_{01}}{2\sigma^2} \right) \cdot \right.\right. \tag{3.5}$$

$$\left.\left. I_0 \left(\frac{\frac{V_0}{2\alpha}\sqrt{S_{01}}}{2\sigma^2} \right) dV_0 \right\} d\theta \right] f_{V_1}(V_1|H_1\;)\; dV_1$$

where the order of the integration has been changed for computational convenience. In the above equation the inner-most integral can be expressed in terms of the Marcum Q-function as follows

$$\int_{V_1}^\infty \frac{V_0}{\sigma^2} \exp\left(- \frac{\left(\frac{V_0}{2\alpha}\right)^2 + S_{01}}{2\sigma^2} \right) I_0 \left(\frac{\frac{V_0}{2\alpha}\sqrt{S_{01}}}{\sigma^2} \right) dV_0$$

$$= Q\left(\frac{\sqrt{S_{01}}}{\sigma}, \frac{V_1}{2\alpha\sigma} \right)$$

Then, Equation 3.5 for Pe becomes

$$P_{e_1} = \frac{1}{2\pi} \int_0^{2\pi} \left[\int_0^{2\pi} Q\left(\frac{\sqrt{S_{01}}}{\sigma}, \frac{V_1}{2\alpha\sigma} \right) d\theta \right] f_{V_1}(V_1|H_1)\; dV_1 \tag{3.6}$$

$$= \int_0^\infty \left[\frac{1}{2\pi} \int_0^{2\pi} Q\left(\frac{\sqrt{S_{01}}}{\sigma}, \frac{1-\alpha}{\alpha} x \right) d\theta \right] \cdot$$

$$\left[\frac{1}{2\pi} \int_0^{2\pi} x \exp\left(- \frac{x^2 + \left(\frac{\sqrt{S_{11}}}{\sigma}\right)^2}{2} \right) I_0\left(x \frac{\sqrt{S_{11}}}{\sigma} \right) d\theta \right] dx$$

where in the second equality the change of variable $x = \frac{y}{2(1-\alpha)\sigma}$, has been made.

From the orthogonality property of the signal pair used (which is obtained by assuming sufficient separation between two frequencies and that W_1 as well as W_0 are large), we have

$$\int_0^T \sin(W_1 t + \theta)\sin W_0 t \, dt = 0$$

so that the term S_{01} is independent of θ. Therefore Equation 3.6 can be rewritten in the following form

$$P_{e_1} = \frac{1}{2\pi}\int_0^{2\pi}\left[\int_0^\infty Q\left(\frac{\sqrt{S_{01}}}{\sigma}, \frac{1-\alpha}{\alpha}x\right) x \exp\left(-\frac{x^2 + \left(\frac{\sqrt{S_{11}}}{\sigma}\right)^2}{2}\right) \cdot \right. \tag{3.7}$$

$$\left. I_0\left(x \frac{\sqrt{S_{11}}}{\sigma}\right) dx\right] d\theta$$

where the order of integration has been changed. Furthermore, using the following formula involving an integral of a Marcum Q-function [Ref. 4]

$$\int_0^\infty Q\left(\frac{\alpha_2}{\sigma_2}, \frac{R_1}{\sigma_2}\right)\frac{R_1}{\sigma_1^2}\exp\left(-\frac{\alpha_1^2 + R_1^2}{2\sigma_1^2}\right) I_0\left(\frac{\alpha_1 R_1}{\sigma_1^2}\right) dR_1$$

$$= \frac{\sigma_1^2}{\sigma_1^2+\sigma_2^2}\left[1 - Q\left(\sqrt{\frac{\alpha_1^2}{\sigma_1^2+\sigma_2^2}}, \sqrt{\frac{\alpha_2^2}{\sigma_1^2+\sigma_2^2}}\right)\right]$$

$$+ \frac{\sigma_2^2}{\sigma_1^2+\sigma_2^2} Q\left(\sqrt{\frac{\alpha_2^2}{\sigma_1^2+\sigma_2^2}}, \sqrt{\frac{\alpha_1^2}{\sigma_1^2+\sigma_2^2}}\right)$$

the inner integral in Equation 3.7 can be simplified in such a way that Pe_1 becomes

$$P_{e_1} = \frac{(1-\alpha)^2}{(1-\alpha)^2+\alpha^2}\left[1 - \frac{1}{2\pi}\int_0^{2\pi} Q\left(\frac{\sqrt{S_{11}}}{\sigma}\sqrt{\frac{(1-\alpha)^2}{(1-\alpha)^2+\alpha^2}}, \frac{\sqrt{S_{01}}}{\sigma}\sqrt{\frac{\alpha^2}{(1-\alpha)^2+\alpha^2}}\right) d\theta\right] \tag{3.8}$$

$$I_0\left(x\frac{\sqrt{S_{11}}}{\sigma}\right)dx\Big\}\,d\theta\Big]\frac{A}{A_0^2}\exp\left(-\frac{A^2}{2A_0^2}\right)dA$$

$$=\frac{1}{2\pi}\int_0^{2\pi}\left[\int_0^{\infty}Q\left(\frac{\sqrt{S_{11}}}{\sigma},x\right)x\exp\left(-\frac{x^2}{2}\right)\right.$$

$$\left.\left\{\frac{A}{A_0^2}\exp\left[-\frac{\left(\frac{A_0}{\sigma}\sqrt{S_{11}}\right)^2-A^2}{2A}\right]I_0\left(x\frac{\sqrt{S_{11}}}{\sigma}\right)dA\right\}dx\right]d\theta$$

where the last equality has been obtained by interchanging the order of integration. This equation does not appear to be readily simplifiable. If we take a closer look at the innermost integral in Equation 4.1, that integral might be evaluated using

$$\int_0^{\infty}t^{u-1}\,I_v(\alpha\,t)\exp(-p^2t^2)\,dt$$

$$=\frac{\Gamma\left(\frac{u+v}{2}\right)\left(\frac{\alpha}{2p}\right)^{v}}{2p^{u}\,\Gamma(v+1)}\exp\left(\frac{\alpha^2}{4p^2}\right){}_1F_1\left(\frac{v-u}{2}+1,\ v+1;\ \frac{-\alpha}{4p^2}\right)$$

where $I_v(\cdot)$ is the modified Bessel function of the first kind of order v, $\Gamma(\cdot)$ is the Gamma function, and ${}_1F_1(X_1,X_2;X_3)$ is the Confluent Hypergeometric function. In the special case of $X_1 = 0$, the Confluent Hypergoemetric function ${}_1F_1(0,X_2;X_3) = 1$ so that the integral in question can be simplified in this case. However, since the term S_{11} includes the integration factor A, we may be able to calculate and express Equation 4.1 in a simpler form using the formula above for the case in which the variable A could be separated out in the term $\sqrt{S_{11}}$ for a given jammer waveform.

The same arguments apply to obtaining the probability of error under the hypotheses H_0. Thus Pe can be shown to be given by

$$P_{e_0}=\frac{1}{2\pi}\int_0^{2\pi}\left[\int_0^{\infty}Q\left(\frac{\sqrt{S_{10}}}{\sigma},x\right)x\exp\left(-\frac{x^2}{2}\right)\right. \tag{4.2}$$

$$\left.\left\{\int_0^{\infty}\frac{A}{A_0^2}\exp\left[-\frac{\left(\frac{A_0}{\sigma}\sqrt{S_{00}}\right)^2-A^2}{2A_0^2}\right]I_0\left(x\frac{\sqrt{S_{00}}}{\sigma}\right)dA\right\}dx\right]d\theta$$

$$H_1 : \; r(t) = A \sin(W_1 t + \Theta) + n(t) + n_J(t) \quad 0 \leq t \leq T$$

or

$$H_o : \; r(t) = B \sin(W_o t + \Phi) + n(t) + n_J(t) \quad 0 \leq t \leq T$$

where the amplitudes A and B are independent identically distributed random variables having density functions

$$f_A(a) = \frac{a}{A_o^2} \exp\left(-\frac{a^2}{2A_o^2}\right) \qquad a \geq 0$$

$$f_B(b) = \frac{b}{A_o^2} \exp\left(-\frac{b^2}{2A_o^2}\right) \qquad b \geq 0$$

with $E\{A\} = E\{B\} = \sqrt{\frac{\pi}{2}}\, A_o$ and $E\{A^2\} = E\{B^2\} = 2A_o^2$.

Note that FSK with fading is the same as what was previously analyzed (FSK without fading) except that now the signal amplitudes are random varibles. Therefore for a fixed value of A, the error probability when a "mark" signal has been transmitted is the same as that of the noncoherent (nonfading) FSK case. That is, we can use the results of the preceding chapter and in particular make use of Equation 3.7, to obtain (for the special case of $\alpha = 1/2$)

$$P_{e_1}(A) = \frac{1}{2\pi} \int_0^{2\pi} \left[\int_0^{\infty} Q\left(\frac{\sqrt{S_{o1}}}{\sigma}, x\right) x \exp\left(-\frac{x^2 + \left(\frac{\sqrt{S_{11}}}{\sigma}\right)^2}{2}\right) I_o\left(x \frac{\sqrt{S_{11}}}{\sigma}\right) dx \right] d\theta$$

where the dependence on A is due to the fact that the term S_{11} defined previously depends on A. Then, P_{e_1} becomes

$$P_{e_1} = \int_0^{\infty} P_{e_1}(A)\, P(A)\, dA$$

$$= \frac{1}{2\pi} \int_0^{\infty} \left[\int_0^{2\pi} \left\{ \int_0^{\infty} Q\left(\frac{\sqrt{S_{o1}}}{\sigma}, x\right) x \exp\left(-\frac{x^2 + \left(\frac{\sqrt{S_{11}}}{\sigma}\right)^2}{2}\right) \cdot \right.\right. \tag{4.1}$$

IV. ANALYSIS OF FSK IN THE PRESENCE OF JAMMING AND FADING

A. GENERAL

In certain propagation media, the received signal transmitted via a free space channel is often subject to fading, a phenomenon caused by multipath propagation and the equivalent addition of random phasors. The vector sum signal will have an envelope which changes with time resulting in an effect known as fading.

The fading signal model often utilized is that of the nonselective, slow fading, Rayleigh distributed signal amplitude where it is assumed that the amplitude, while random, remains constant over the time interval [0,T]. Thus the received envelope is now random with probability density function

$$f_A(a) = \frac{2a}{\bar{E}} \exp\left(-\frac{a^2}{\bar{E}}\right) \qquad a \geq 0$$

where $\bar{E} = E(A^2)$ denotes the mean squared value of the signal amplitude.

The optimum (minimum probability of error) receiver structure in the presence of additive white Gaussian noise (however with no jamming present) is the same as that for noncoherent nonfading FSK and its structure is therefore given by the receiver of Figure 5.

B. ANALYSIS WITH NEAR OPTIMUM JAMMER

We shall analyze the effect of Rayleigh fading on an incoherent FSK receiver operating in the presence of a jammer in addition to the additive white Gaussian noise. Thus, the signals at the front end of the receiver are either

C_1 and C_2 have been defined in Equation 3.13. As before, in this equation SNR and JSR are defined by Equations 2.10 and 2.11 respectively.

The performance results to be presented are a function of SNR, JSR, and the number of times the jammer sweeps the signal band. This is discussed in greater detail in Chapter 5.

We note that all the terms in Equation 3.24 and Equation 3.25 take on the form of a SINC function times a sine function. We can readily show that for even values of m and l,

$$(n_j, C)_1 = (n_j, C)_0 = 0 \tag{3.26}$$

Using the mathematical forms given by Equations 3.22, 3.23 and 3.26, the performance of the receiver with FM sweep jamming is obtained via the use of Equations 3.8 and 3.9 and evaluated from the expression

$$\begin{aligned} P_e = {} & P(H_1)\, C_1 \left[1 - \frac{1}{2\pi} \int_0^{2\pi} Q\left(\sqrt{C_1 \alpha_{11}}, \sqrt{C_2 \beta_{01}}\right) d\theta \right] \\ & + P(H_1)\, C_2 \left[\frac{1}{2\pi} \int_0^{2\pi} Q\left(\sqrt{C_2 \beta_{01}}, \sqrt{C_1 \alpha_{11}}\right) d\theta \right] \\ & + P(H_0)\, c_2 \left[1 - \frac{1}{2\pi} \int_0^{2\pi} Q\left(\sqrt{C_2 \alpha_{00}}, \sqrt{C_1 \beta_{10}}\right) d\phi \right] \\ & + P(H_0)\, c_1 \left[\frac{1}{2\pi} \int_0^{2\pi} Q\left(\sqrt{C_1 \beta_{10}}, \sqrt{C_2 \alpha_{00}}\right) d\phi \right] \end{aligned} \tag{3.27}$$

where

$$\alpha_{11} = 2\,\mathrm{SNR} \left[1 + 2\sqrt{\mathrm{JSR}}\, J_{n_1}(\beta) \cos\theta + \mathrm{JSR}\, J_{n_1}^2(\beta) \right]$$

$$\alpha_{00} = 2\,\mathrm{SNR} \left[1 + 2\sqrt{\mathrm{JSR}}\, J_{n_3}(\beta) \cos\phi + \mathrm{JSR}\, J_{n_3}^2(\beta) \right]$$

$$\beta_{01} = \beta_{10} = 2\,\mathrm{SNR}\,\mathrm{JSR}\, J_{n_1}^2(\beta)$$

$J_K(\beta) \ll 1$ for $\beta \ll k$. Hence the terms $J_{n_2}(\beta)$ and $J_{n_4}(\beta)$ can be neglected so that

$$(n_j,S)_1 = \sqrt{\frac{P_{n_j}T}{2}}\, J_{n_1}(\beta) \tag{3.22}$$

and

$$(n_j,S)_0 = \sqrt{\frac{P_{n_j}T}{2}}\, J_{n_3}(\beta) \tag{3.23}$$

The other parameters to be considered are

$$(n_j,C)_K = \int_0^T \sqrt{P_{n_j}}\sqrt{\frac{2}{T}} \sin(W_s t + \beta \sin W_j t) \cos W_K t \, dt$$

where $\beta = K_f a / W_j$. Using again the Bessel function coefficient expansion and appropriate trigonometric identities, we obtain

$$(n_j,C)_1 = \sqrt{\frac{P_{n_j}T}{2}} \sum_{n=-\infty}^{\infty} J_n(\beta) \left[\frac{\sin^2 \frac{\pi}{2}(2nK+m+\ell/2)}{\frac{\pi}{2}(2nK+m+\ell/2)} + \frac{\sin^2 \frac{\pi}{2}(2nK-\ell/2)}{\frac{\pi}{2}(2nK-\ell/2)} \right. \tag{3.24}$$

and

$$(n_j, C)_0 = \sqrt{\frac{P_{n_j}T}{2}} \sum_{n=-\infty}^{\infty} J_n(\beta) \left[\frac{\sin^2 \frac{\pi}{2}(2nk+1/2)}{(2nk+1/2)} + \frac{\sin^2 \frac{\pi}{2}(2nk+m-1/2)}{(2nk+m-1/2)} \right] \tag{3.25}$$

$$(n_j,S)_1 = \sqrt{\frac{P_{n_j}T}{2}}\sum_{n=-\infty}^{\infty} J_n(\beta)\Bigg[\frac{\sin\pi(2nk-\ell/2)}{\pi(2nk-\ell/2)} - \frac{\sin\pi(2nk+m+\ell/2)}{\pi(2nk+m+\ell/2)}\Bigg] \qquad (3.20)$$

and

$$(n_j,S)_0 = \sqrt{\frac{P_{n_j}}{2T}}\sum_{n=-\infty}^{\infty} J_n(\beta)\Bigg[\frac{\sin\left[\frac{1}{2}(\omega_1-\omega_0)+n\omega_j\right]T}{\frac{1}{2}(\omega_1-\omega_0)+n\omega_j} - \frac{\sin\left[\frac{1}{2}(\omega_1+3\omega_0)+n\omega_j\right]T}{\frac{1}{2}(\omega_1+3\omega_0)+n\omega_j}\Bigg]$$

which becomes

$$(n_j,S)_0 = \sqrt{\frac{P_{n_j}T}{2}}\sum_{n=-\infty}^{\infty} J_n(\beta)\Bigg[\frac{\sin\pi(2nk+\ell/2)}{\pi(2nk+\ell/2)} - \frac{\sin\pi(2nk+m-\ell/2)}{\pi(2nk+m-\ell/2)}\Bigg] \qquad (3.21)$$

If l is an even integer, then Equation 3.20 and Equation 3.21 at most contain two terms respectively. These terms can exist only if the argument of all SINC functions is zero. Therefore Equation 3.20 and Equation 3.21 can be simplified to yield

$$(n_j,S)_1 = \sqrt{\frac{P_{n_j}T}{2}}\left[J_{n_1}(\beta) - J_{n_2}(\beta)\right]$$

where $n_1 = 1/4k$, $n_2 = -(m + 1/2)/2k$ and

$$(n_j,S)_0 = \sqrt{\frac{P_{n_j}T}{2}}\left[J_{n_3}(\beta) - J_{n_4}(\beta)\right]$$

where $n_3 = -1/4k$, $n_4 = -(m - 1/2)/2k$.

In view of practical communication system constraints, the integer m will typically be much larger than l because m and l represent the sum and difference of the signaling frequencies respectively. In particular we see that

Since the FSK signal covers approximately the band $(W_0 - 4\pi/T\ ,\ W_1 + 4\pi/T)$ (assuming $W_1 > W_0$), it is reasonable to set

$$W_S = \frac{1}{2}(W_1 + w_o) \tag{3.19}$$

so that the instantaneous jammer frequency band $(W_S - \beta W_j\ ,\ W_S + \beta W_j)$ completely covers the signal band provided that

$$W_S - \beta W_j = w_o - \frac{4\pi}{T} \quad ; \qquad W_S + \beta W_j = W_1 + \frac{4\pi}{T}$$

This means that

$$\beta W_j = \frac{1}{2}(W_1 - W_o) + \frac{4\pi}{T}$$

must be satisfied. Since for FSK signaling we have assumed that

$$(W_1 - w_o) T = l\pi \quad ; \qquad (W_1 + w_o) T = m\pi$$

where l and m are integers, and $W_j T = 2\pi k$, it is apparent that

$$\beta = \frac{1}{2K}\left[4 + l/2\right]$$

Note now that the integer k determines the number of times the jamming waveform will sweep the signal band in one bit interval. Thus from Equation 3.18 and Equation 3.19, we have

$$(n_j, S)_1 \quad \sqrt{\frac{P_{n_j}}{2T}} \sum_{n=-\infty}^{\infty} J_n(\beta)\left[\frac{\sin\left[\frac{1}{2}(\omega_o - \omega_1) + n\omega_j\right]T}{\frac{1}{2}(\omega_o - \omega_1) + n\omega_j} - \frac{\sin\left[\frac{1}{2}(3\omega_1 + \omega_o) + n\omega_j\right]T}{\frac{1}{2}(3\omega_1 + \omega_o) + n\omega_j}\right]$$

which becomes

where $\beta = K_f a/W_j$ and θ is a deterministic phase angle. The instantaneous jammer frequency is

$$W_i(t) = W_s + \beta W_j \cos W_j t$$

so that the instantaneous jammer frequency covers the band $(W_s - \beta W_j, \; W_s + \beta W_j)$. Assume that $W_s T = 2\pi l$ and $W_j T = 2\pi k$, where l and k are integers. In order to determine receiver performance in the presence of such a jammer, the parameters S_{ik} $(i,k = 0,1)$ which are a function of $n_j(t)$, must be determined. Before so doing, we evaluate

$$(n_j,S)_K = \int_0^T \sqrt{P_{n_j}}\sqrt{\frac{2}{T}} \sin(W_s t + \beta \sin W_j t) \sin W_K t \, dt \qquad (3.16)$$

$$K = 0, 1$$

where since the fixed phase θ represents a time delay only, it is set to zero for computational ease. Equation 3.16 can be rewritten as follows

$$(n_j,S)_K = \sqrt{\frac{P_{n_j}}{2T}} \int_0^T \Big\{ \cos\left[(W_s - W_K)t + \beta \sin W_j t\right] - \cos\left[(W_s + W_K)t + \beta \sin W_j t\right] \Big\} \, dt \qquad (3.17)$$

By using the well-known Bessel function coefficient expansion for each cosine term in Equation 3.17, the following is obtained

$$(n_j,s)_K = \sqrt{\frac{P_{n_j}}{2T}} \sum_{n=-\infty}^{\infty} J_n(\beta) \left[\frac{\sin[(W_s - W_K) + nW_j]T}{(W_s - W_K) + nW_j} - \frac{\sin[(W_s + W_K) + nW_j]T}{(W_s + W_K) + nW_j} \right] \qquad (3.18)$$

and if the second choice is used, we obtain

$$\begin{aligned} P_e = {} & P(H_1)\, C_1 \left[1 - \frac{1}{2\pi}\int_0^{2\pi} Q\left(\sqrt{C_1 \alpha'_{00}}\ ,\sqrt{C_2 \beta'_{10}}\right) d\theta \right] \\ & + P(H_1)\, C_2 \left[\frac{1}{2\pi}\int_0^{2\pi} Q\left(\sqrt{C_2 \beta'_{10}}\ ,\sqrt{C_1 \alpha'_{00}}\right) d\theta \right] \\ & + P(H_0)\, C_2 \left[1 - \frac{1}{2\pi}\int_0^{2\pi} Q\left(\sqrt{C_2 \alpha'_{11}}\ ,\sqrt{C_1 \beta'_{01}}\right) d\phi \right] \\ & + P(H_0)\, C_1 \left[\frac{1}{2\pi}\int_0^{2\pi} Q\left(\sqrt{C_1 \beta'_{01}}\ ,\sqrt{C_2 \alpha'_{11}}\right) d\phi \right] \end{aligned}$$

The effect of varying the threshold α on receiver performance for the jammer waveform of Equation 3.14 is discussed in Chapter 5.

C. FREQUENCY MODULATION SWEEP JAMMING

In certain situations, the need to jam a certain frequency band rather than a discrete set of frequencies using tone jammers may arise. Therefore an FM sweep jammer will be proposed, analyzed, and its effect on the incoherent FSK receiver investigated as a function of the number of times the jammer sweeps the signal band during the signaling interval [0,T]. The mathematical model used for an FM sweep jammer is

$$n_j(t) = \sqrt{P_{n_{jv}}}\sqrt{\frac{2}{T}}\sin\left[W_s t + K_f \int a \cos W_j t\, dt \right] \qquad 0 \le t \le T$$

After carring out the integration we obtain

$$n_j(t) = \sqrt{P_{n_{jv}}}\sqrt{\frac{2}{T}}\sin\left[W_s t + \beta \sin W_j t + \theta \right]$$

where

$$C_1 = \frac{(1-\alpha)^2}{(1-\alpha)^2+\alpha^2}, \quad C_2 = \frac{\alpha^2}{(1-\alpha)^2+\alpha^2}$$

The effect of a jammer that uses a single tone only to jam either 'mark' or 'space' signals, can be analyzed by setting

$$n_j(t) = \sqrt{P_{n_j}}\sqrt{\frac{2}{T}} \sin W_1 t \tag{3.14}$$

or

$$n_j(t) = \sqrt{P_{n_j}}\sqrt{\frac{2}{T}} \sin W_0 t$$

If the first choice is used, we obtain

$$\begin{aligned} P_e = {} & P(H_1)\, C_1 \left[1 - \frac{1}{2\pi}\int_0^{2\pi} Q\left(\sqrt{C_1 \alpha_{11}'}, \sqrt{C_2 \beta_{01}'}\right) d\theta \right] \\ & + P(H_1)\, C_2 \left[\frac{1}{2\pi}\int_0^{2\pi} Q\left(\sqrt{C_2 \beta_{01}'}, \sqrt{C_1 \alpha_{11}'}\right) d\theta \right] \\ & + P(H_0)\, C_2 \left[1 - \frac{1}{2\pi}\int_0^{2\pi} Q\left(\sqrt{C_2 \alpha_{00}'}, \sqrt{C_1 \beta_{10}'}\right) d\phi \right] \\ & + P(H_0)\, C_1 \left[\frac{1}{2\pi}\int_0^{2\pi} Q\left(\sqrt{C_1 \beta_{10}'}, \sqrt{C_2 \alpha_{00}'}\right) d\phi \right] \end{aligned} \tag{3.15}$$

where

$$\alpha_{11}' = 2\ SNR\ (1 + 2\sqrt{JSR}\cos\theta + JSR)$$

$$\beta_{01}' = 0$$

$$\alpha_{00}' = 2\ SNR$$

$$\beta_{10}' = 2\ SNR\ JSR$$

Thus the probability of error of the unmodified receiver (Equation 3.10) can be expressed in terms of SNR and JSR only (defined by Equations 2.10 and 2.11) as follows

$$P_e = \frac{1}{2}\left[1 - \frac{1}{4\pi}\int_0^{2\pi}\left\{Q\left(\sqrt{\frac{\alpha_{11}}{2}}, \sqrt{\frac{\beta_{01}}{2}}\right) - Q\left(\sqrt{\frac{\beta_{01}}{2}}, \sqrt{\frac{\alpha_{11}}{2}}\right)\right\} d\theta \right. \tag{3.12}$$

$$\left. - \frac{1}{4\pi}\int_0^{2\pi}\left\{Q\left(\sqrt{\frac{\alpha_{00}}{2}}, \sqrt{\frac{\beta_{10}}{2}}\right) - Q\left(\sqrt{\frac{\beta_{10}}{2}}, \sqrt{\frac{\alpha_{00}}{2}}\right)\right\} d\phi\right]$$

where

$$\alpha_{11} = \mathrm{SNR}\,(2 + 2\sqrt{2\mathrm{JSR}}\cos\theta + \mathrm{JSR})$$

$$\alpha_{00} = \mathrm{SNR}\,(2 - 2\sqrt{2\mathrm{JSR}}\cos\phi + \mathrm{JSR})$$

$$\beta_{01} = \beta_{10} = \mathrm{SNR}\ \mathrm{JSR}$$

Receiver performance can now be evaluated as a function of SNR for fixed values of JSR.

To provide further insight into the performance of the modified incoherent receiver, the effect of varying α can be analyzed via computation of the probability of error Pe from Equations 3.8 and 3.9. In terms of SNR and JSR, Pe becomes

$$\begin{aligned} P_e &= P(H_1)\, P_{e_1} + P(H_0)\, P_{e_0} \\ &= P(H_1)\, C_1\left[1 - \frac{1}{2\pi}\int_0^{2\pi} Q\left(\sqrt{C_1\alpha_{11}}, \sqrt{C_2\beta_{01}}\right) d\theta\right] \\ &\quad + P(H_1)\, C_2\left[\frac{1}{2\pi}\int_0^{2\pi} Q\left(\sqrt{C_2\beta_{01}}, \sqrt{C_1\alpha_{11}}\right) d\theta\right] \\ &\quad + P(H_0)\, C_2\left[1 - \frac{1}{2\pi}\int_0^{2\pi} Q\left(\sqrt{C_2\alpha_{00}}, \sqrt{C_1\beta_{10}}\right) d\phi\right] \\ &\quad + P(H_0)\, C_1\left[\frac{1}{2\pi}\int_0^{2\pi} Q\left(\sqrt{C_1\beta_{10}}, \sqrt{C_2\alpha_{00}}\right) d\phi\right] \end{aligned} \tag{3.13}$$

Observe that Equation 3.10 yields the performance of a conventional incoherent FSK receiver (with no channel weighting) in the presence of jamming. If we now use as the jamming waveform the jammer which is optimum against a coherent FSK receiver, that is

$$n_j(t) = \sqrt{P_{n_j}}\frac{2}{\sqrt{T}}\sin\frac{1}{2}(W_1 - W_0)t\cos\frac{1}{2}(W_1 + W_0)t \qquad (3.11)$$
$$= \sqrt{\frac{P_{n_j}}{T}}\left[\sin W_1 t - \sin W_0 t\right]$$

then the terms S_{ik} $(i,k=0,1)$ in Equations 3.8 and 3.9, which are a function of the jammer waveform $n_j(t)$, can be computed as follows

$$S_{11} = \left[(S_1,S)_1 + (n_j,S)_1\right]^2 + \left[(S_1,C)_1 + (n_j,c)_1\right]^2$$
$$= \left(\frac{AT}{2}\right)^2 + \frac{AT\sqrt{P_{n_j}T}}{2}\cos\theta + \frac{P_{n_j}T}{4}$$

$$S_{01} = \left[(S_1,S)_0 + (n_j,C)_0\right]^2 + \left[(S_1,C)_0 + (n_j,c)_0\right]^2$$
$$= \frac{P_{n_j}T}{4}$$

and

$$S_{00} = \left[(S_0,S)_0 + (n_j,C)_0\right]^2 + \left[(S_0,C)_0 + (n_j,C)_0\right]^2$$
$$= \left(\frac{AT}{2}\right)^2 - \frac{AT\sqrt{P_{n_j}T}}{2}\cos\phi + \frac{P_{n_j}T}{4}$$

$$S_{10} = \left[(S_0,S)_1 + (n_j,S)_1\right]^2 + \left[(S_0,C)_1 + (n_j,C)\right]^2$$
$$= \frac{P_{n_j}T}{4}$$

$$+ \frac{\alpha^2}{(1-\alpha)^2+\alpha^2}\left[\frac{1}{2\pi}\int_0^{2\pi} Q\left(\frac{\sqrt{S_{01}}}{\sigma}\sqrt{\frac{\alpha^2}{(1-\alpha)^2+\alpha^2}},\ \frac{\sqrt{S_{11}}}{\sigma}\sqrt{\frac{(1-\alpha)^2}{(1-\alpha)^2+\alpha^2}}\right) d\theta\right]$$

where the dependence on θ is imbedded in the term S_{11}.

Following exactly the same procedure used in obtaining the expression for Pe_1, it can be established that the expression for Pe_0, which denotes the error probability when H_0 is assumed, that is, Prob.$(V_1 > V_0 \mid H_0)$ takes the form

$$P_{e_0} = \frac{\alpha^2}{(1-\alpha)^2+\alpha^2}\left[1 - \frac{1}{2\pi}\int_0^{2\pi} Q\left(\frac{\sqrt{S_{00}}}{\sigma}\sqrt{\frac{(1-\alpha)^2}{(1-\alpha)^2+\alpha^2}},\ \frac{\sqrt{S_{01}}}{\sigma}\sqrt{\frac{(1-\alpha)^2}{(1-\alpha)^2+\alpha^2}}\right) d\phi\right] \tag{3.9}$$

$$+ \frac{(1-\alpha)^2}{(1-\alpha)^2+\alpha^2}\left[\frac{1}{2\pi}\int_0^{2\pi} Q\left(\frac{\sqrt{S_{01}}}{\sigma}\sqrt{\frac{(1-\alpha)^2}{(1-\alpha)^2+\alpha^2}},\ \frac{\sqrt{S_{00}}}{\sigma}\sqrt{\frac{\alpha^2}{(1-\alpha)^2+\alpha^2}}\right) d\phi\right]$$

where only the term S_{00} is dependent on ϕ. Therefore the total average probability of error (Pe) can be obtained from

$$P_e = \frac{1}{2}(P_{e_1} + P_{e_0})$$

assuming that the two hypotheses are equally likely.

For the special case $\alpha = 1/2$, the performance of the optimum receiver in the presence of jamming is given by

$$P_e(\alpha = \tfrac{1}{2}) = \frac{1}{2}\left[1 - \frac{1}{2\pi}\int_0^{2\pi}\left\{Q\left(\frac{1}{\sqrt{2}}\frac{\sqrt{S_{11}}}{\sigma}, \frac{1}{\sqrt{2}}\frac{\sqrt{S_{01}}}{\sigma}\right)\right.\right. \tag{3.10}$$

$$\left. - Q\left(\frac{1}{\sqrt{2}}\frac{\sqrt{S_{01}}}{\sigma}, \frac{1}{\sqrt{2}}\frac{\sqrt{S_{11}}}{\sigma}\right)\right\} d\theta$$

$$- \frac{1}{2\pi}\int_0^{2\pi}\left\{Q\left(\frac{1}{\sqrt{2}}\frac{\sqrt{S_{00}}}{\sigma}, \frac{1}{\sqrt{2}}\frac{\sqrt{S_{10}}}{\sigma}\right)\right.$$

$$\left.\left. - Q\left(\frac{1}{\sqrt{2}}\frac{\sqrt{S_{10}}}{\sigma}, \frac{1}{\sqrt{2}}\frac{\sqrt{S_{00}}}{\sigma}\right)\right\} d\phi\right]$$

Using Equations 4.1 and 4.2, we can evaluate the performance of receiver in a Rayleigh fading environment from

$$P_e = \frac{1}{2}(P_{e1} + P_{eo})$$

assuming that the two signals are equally likely to be sent.

V. DESCRIPTION OF GRAPHICAL RESULTS

A. GENERAL

In this chapter, the analytical results of the previous chapters are now presented via graphical means based on the derived mathematical expressions for receiver probability of error.

The plots presented display the receiver probability of error (Pe) as a function of SNR for the various jammer waveforms previously considered for a set of JSR values.

In each plot, the case JSR = 0 has been included in order to provide the basis for comparisons of the jammer effectiveness on the receiver performamce as it relates to additive white Gaussian noise only interference.

B. ASK (ON - OFF KEYING)

The graphical results for the incoherent ASK receiver performamce are presented first. These plots correspond to numerical evaluations of Equations 2.5 and 2.7 for equally likely hypothesis, that is, $P(H_1)$ and $P(H_0)$ are equal to 1/2.

The plot of Pe for ASK modulation is shown in Figure 6.7 as a function of SNR for fixed values of JSR, using a jammer as specified in Equation 2.6 . Figure 6.7 clearly shows the 'break point' phenomenon in which if JSR increases beyond a certain value (0.25 in this figure), Pe increases with increasing SNR. From this figure, one can observe that 16.0 db of SNR is required to obtain a Pe of 10^{-5} without jammer, i.e., at a JSR value of 0.0. In comparison, it takes 23.5 db of SNR to obtain the same Pe for a JSR value of 0.1. Thus, in the presence of a jammer with a JSR value below the

break point, we need a larger SNR in order to obtain the same performance level of a receiver operating without a jammer interference. However, in a jamming environment, a JSR value above the break point produces a Pe which increases with increasing SNR. In fact, there is no value of SNR that can produce Pe of 10^{-5} for a JSR above the break point. For the case of ASK modulation, the break point occurs at a value of JSR which is approximately 0.25, as obtained from Equations 2.8 and 2.9.

Figure 6.8 shows the variable thresholding effect on the jamming situation with JSR = 0.3 beyond the break point (JSR = 0.25). Instead of using the fixed threshold as given by Equation 2.5, the variation of the receiver threshold obtained from Equation 2.5 by changing the value of R , can reduce the jamming effect over a restricted range of SNR values as shown. However, since as shown in Equation 2.13 the variation of the value R does not significantly affect the threshold value, the variation over a wide range of values of R does not result in a significant change in Pe.

C. FSK WITH TONE JAMMER

This section presents graphical results pertaining to jamming effects on FSK modulation with a single tone jammer acting against one of the two channels and a jammer consisting of two different tones acting against both channels simultaneously.

Figure 6.9 corresponds to the performance of the optimum FSK receiver in which α = 1/2. Equation 3.10 or Equation 3.12 is used to evaluate performance with the near optimum jammer specified in Equation 3.11 . This jammer waveform can be thought of as 'mark' and 'space' channel jamming. Figure 6.9 shows a similar result to that found in the ASK case except that the breakpoint occurs at a higher value of

JSR than that found for ASK. This breakpoint occurs at a JSR somewhere between 0.5 and 1.0 as shown in this plot. From this figure it can be noted that 13.5 db of SNR is need to obtain a Pe of 10^{-5} for a JSR value of 0.0, but the same Pe is obtained by increasing the SNR to 16.5 db for a JSR of 0.1. This demonstrates that relatively low JSR values require a significant SNR boost in order to maintain a certain desired Pe value.

As shown in Figures 6.7 and 6.9, comparison of ASK and FSK modulation reveals that FSK is somewhat less vulnerable to jamming. However it must be remembered that the jammer waveform $n_j(t)$ used in each case is different.

The effect of the single tone jammer on the optimum FSK receiver is presented in Figure 6.10 which corresponds to the evaluation of Equation 3.15 . The single channel jamming on either the 'mark' or the 'space' channel has the same effect insofar as single tone jamming is concerned. Therefore the effect of 'mark' channel jamming only is evaluated and plotted. Note that in Figure 6.10, 19.5 db of SNR is required to obtain a Pe of 10^{-5} as compared to an SNR of 24.5 db in Figure 6.9 for a JSR value of 0.3 with Pe = 10^{-5} also.

As expected, Figures 9 and 10 demonstrate the fact that single channel jamming is less effective than simultaneous jamming of 'space' and 'mark' channels with a near optimum jammer.

The effect of a variable threshold on FSK will be considered by changing the value of α in the modified receiver shown in Figure 6.6 . For a value of α other than 1/2, the simultaneous jamming of 'mark' and 'space' channel results in a compensation of the other channel such that the jamming effect remains the same as in the case of α = 1/2. In other words, it is difficult to reduce the near optimum jamming effect by means of a varying the threshold.

On the other hand, when one channel jamming is applied to the receiver, the jamming effect can be reduced by adjusting the threshold with increasing SNR as shown in Figure 6.11, particularily for JSR of 1.0 and various values of α. A moment's reflection will reveal that the 'mark' channel jammer increases the output power level of the upper envelope detector (see Figure 6.6) so that when the 'space' signal is sent, the error increases. Therefore, for this type of error, under the assumption that the 'space' signal has been transmitted, Pe can be reduced by lowering the level of the output of the multiplier by using an appropriate value of α.

D. FSK WITH FM JAMMER

This section presents the effect of an FM sweep jammer using sinusodal modulation on noncoherent FSK signaling. The FM jammer was designed to sweep the bandwidth occupied by the signal several times during a bit interval. Thus the jammer effectiveness was investigated as a function of the number of times the jammer sweeps over the band of the signal during one bit-time interval.

Figure 6.12 shows the result for one sweep of the jammer per bit interval. Figure 6.13 shows the result of increasing the sweeping to two sweeps per bit interval. These plots show that the FM sweep jammer can be more effective by increasing the number of times of jammer sweeping during a bit interval. This can be expected from the results obtained in Equation 3.27 . In Figure 6.12, in order to obtain a Pe of 10^{-5}, 16.0 db of SNR is required for JSR value of 0.3, but in Figure 6.13 which corresponds to twice the jammer sweeping, the same Pe can be obtained by increasing the SNR value to 19.5 db for the same JSR value of 0.3. In comparison to the previous case of FSK with tone jamming,

for the same jamming environment (i.e., JSR value of 0.3) Figures 6.9 and 6.10 show that 24.5 db and 19.5 db of SNR are required respectively in order to get the same Pe of 10^{-5}.

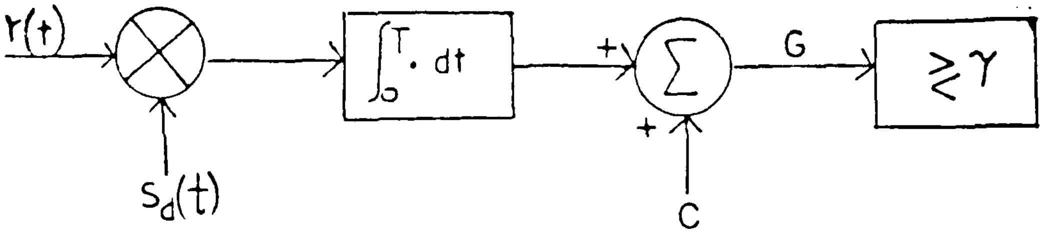
These different requirements of SNR value for various jammer waveforms show that the FM sweep jammer can be effective but in general is not as effective as the near optimum jammer. Note that from a practical point of view, the added complexity of FM jammer waveform may make it an unlikely candidate for a replacement of the near optimum jammer. However, one advantage the FM sweep jammer has over the near optimum jammer is that the former can spread its power over a large bandwidth easily and therefore is more effective than the latter in the case of lack of exact information or knowledge about the signal carrier frequency.

VI. CONCLUSIONS

The familiar model in which Gaussian noise is the total interference is not adequate when jamming or interference signals are present in the transmission environment. This thesis has analyzed the effect of various deterministic jammer waveforms in terms of probability of error on binary incoherent receivers operating in the presence of noise.

From the jammer point of view, the goal is to cause the maximum possible error to the various receivers while making efficient use of its available power (i.e., with fixed jammer power). For coherent receivers, it was proved that the optimum jammer waveform is made of a deterministic signal proportional to the difference of the binary signals used to carry the digital information. This thesis has demonstrated that those optimum jammers derived for coherent receivers perform their function as near optimum jammers satisfactorily against incoherent receivers.

Therefore, these near optimum jammers can be concluded to be one of the most attractive candidates for efficient jamming of binary incoherent communication systems. An optimum jammer has not been derived or analyzed because the complexity of the expression for Pe makes it very difficult if not impossible to derive the optimum jammer waveform in closed form. Other jamming waveforms such as single channel jamming and FM sweep jamming showed its inferiority in comparison to near optimum jammer waveforms and their efficiency can be reduced by means of appropriate variations of the receiver threshold.



$$S_d(t) = S_1 + S_0(t)$$

$$C = \text{bias} = \frac{1}{2} \int_0^T \left[S_0^2(t) - S_1^2(t) \right] dt$$

$$\gamma = \frac{N_0}{2} \ln R \quad ; \quad R = \frac{P\{S_0\}}{P\{S1\}}$$

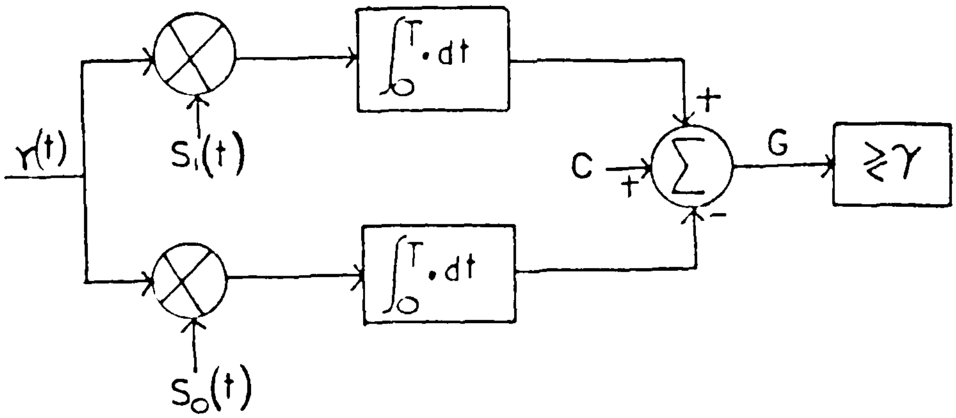
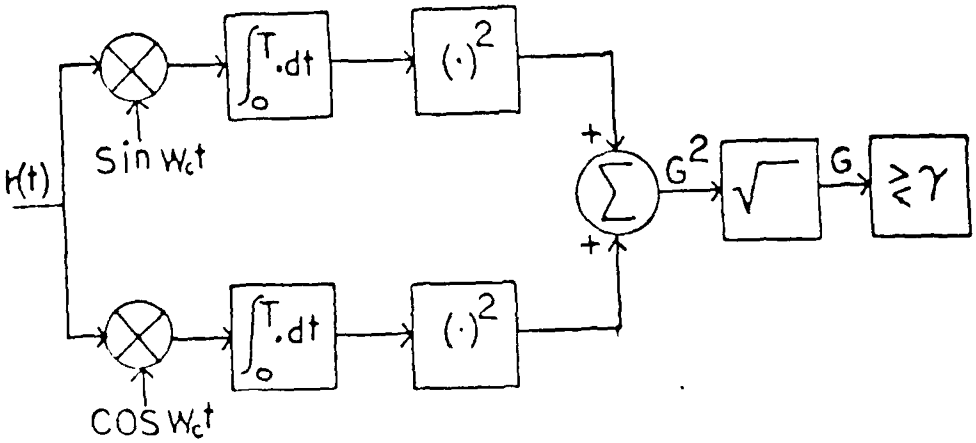


Figure 6.1 Correlation Receiver for Binary Signals.



$$e^{-A^2T/2N_0}\, I_0(2An/N_0) = R$$

Figure 6.2 Quadrature Receiver for ASK.

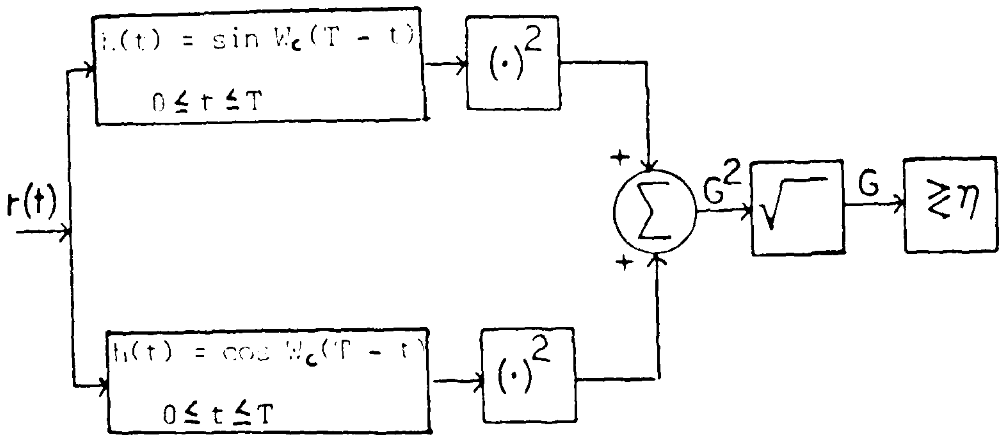


Figure 6.3 Alternate Form of Quadrature Receiver.

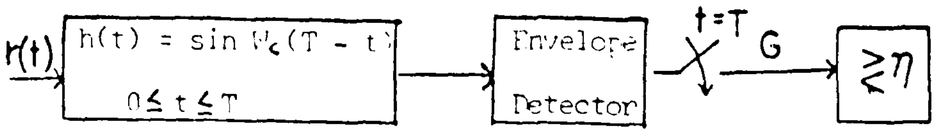


Figure 6.4 Incoherent Matched Filter Receiver.

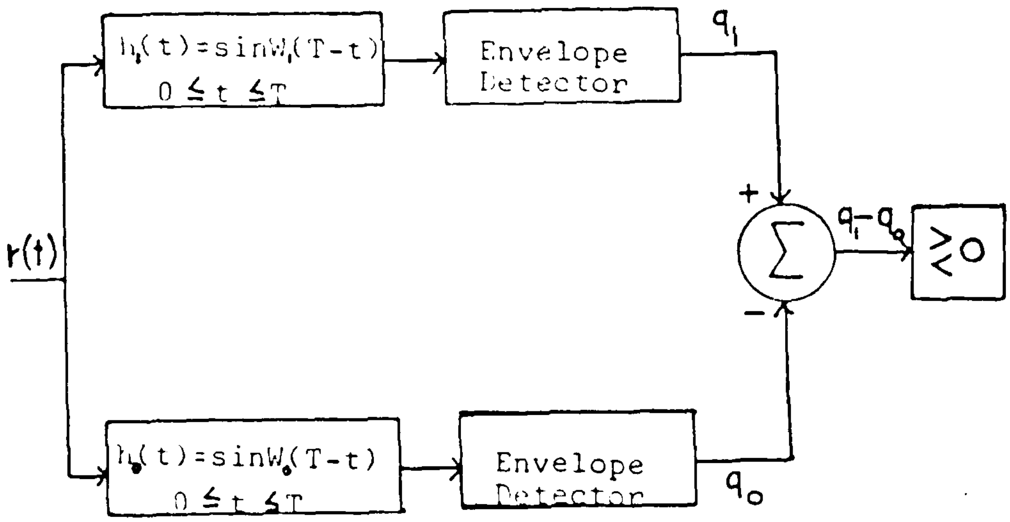


Figure 6.5 Incoherent Frequency Shift Keying (FSK) Receiver.

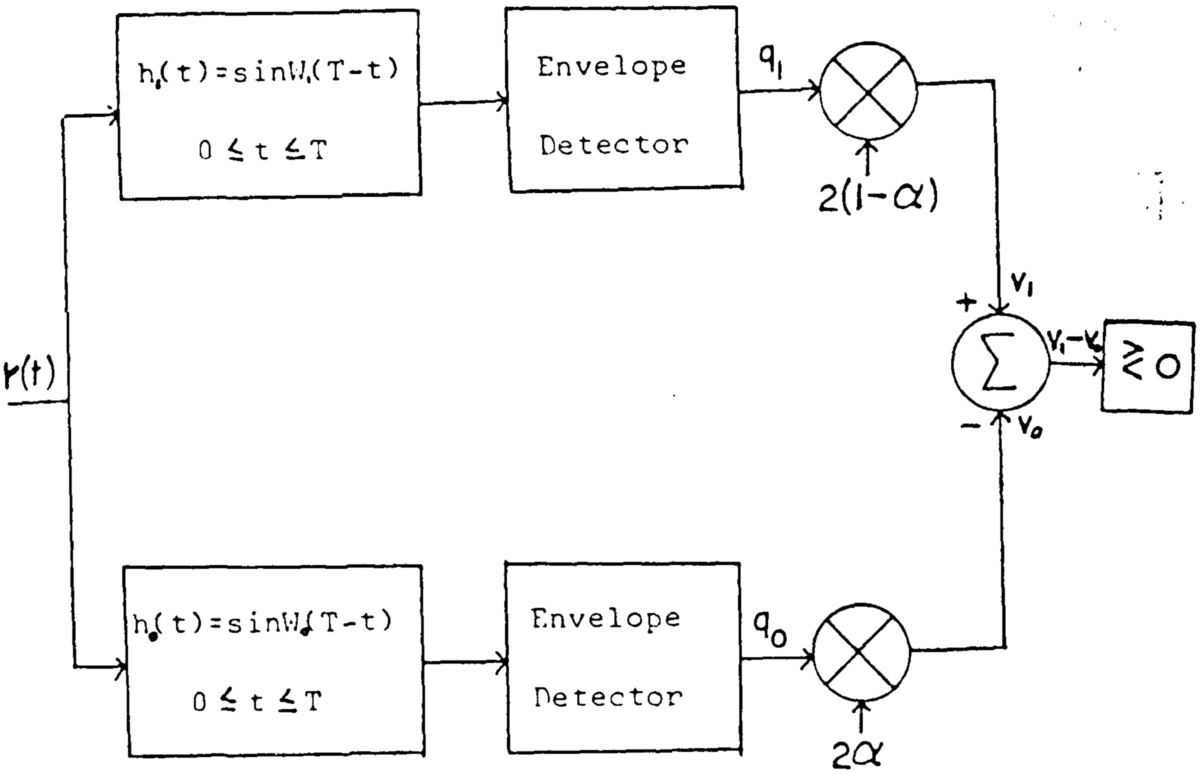


Figure 6.6 Modified Incoherent FSK Receiver.

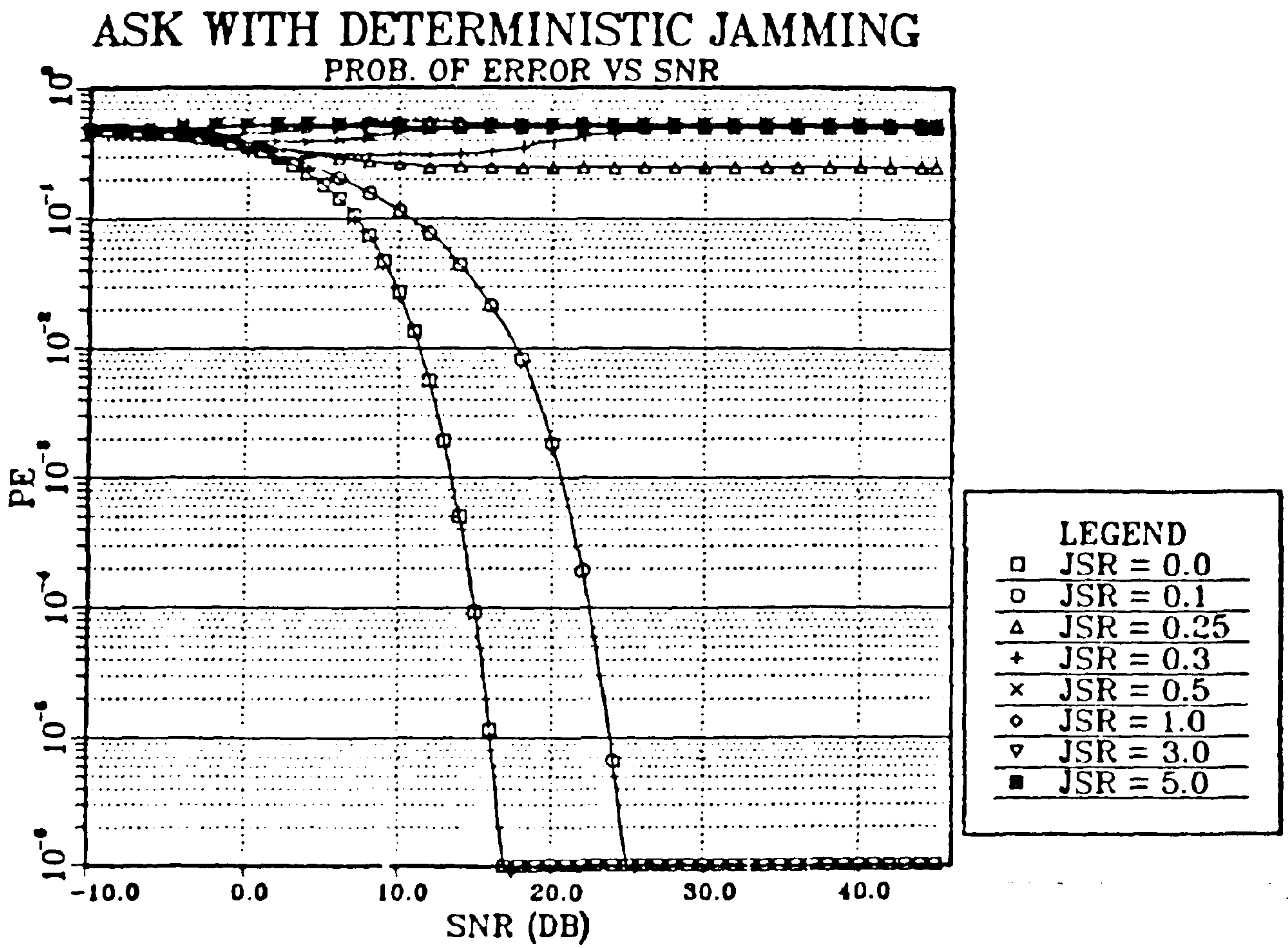


Figure 6.7 Near Optimum Jammer for ASK Modulation.

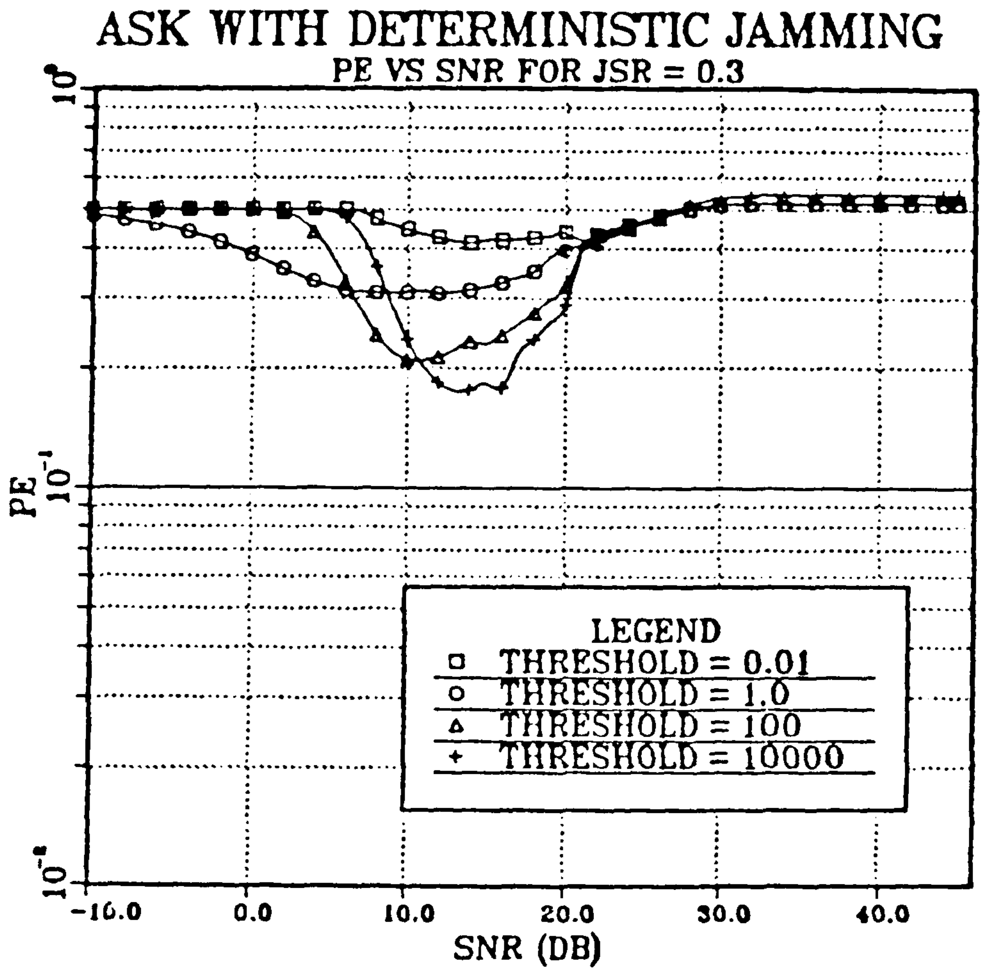


Figure 6.8 Variable Threshold Effect on ASK Modulation.

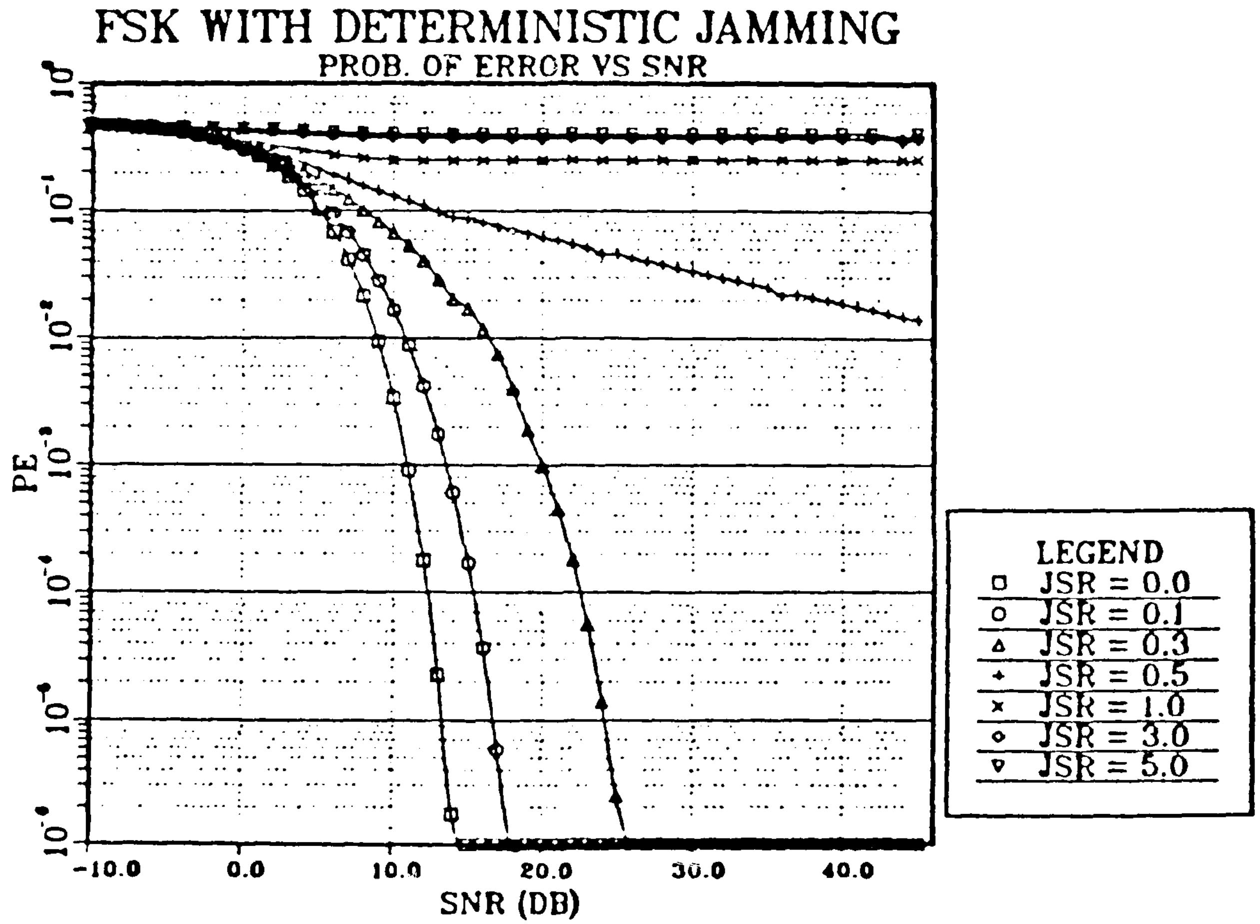


Figure 6.9 Near Optimum Jammer for FSK Modulation.

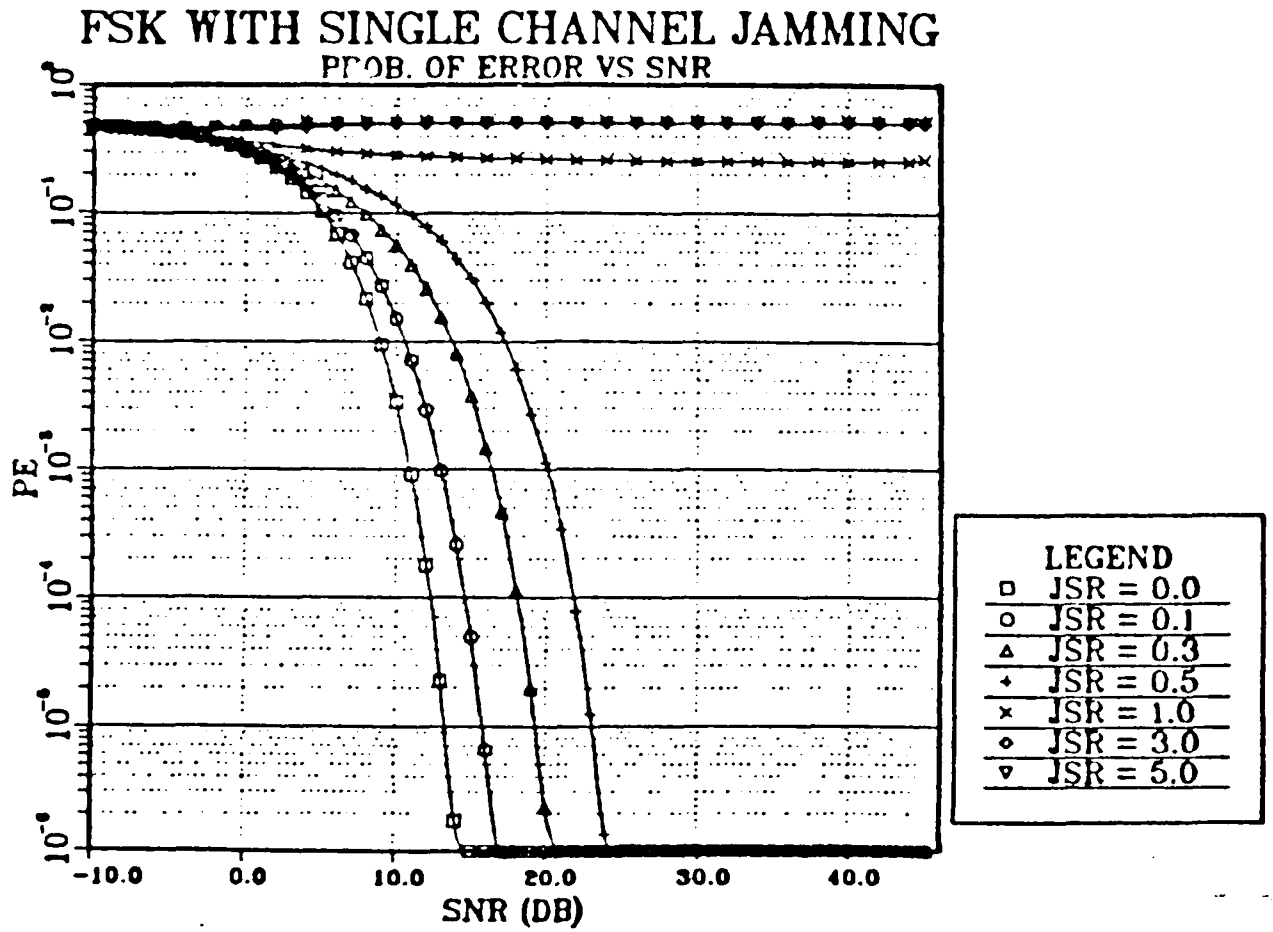


Figure 6.10 Single Channel Jamming for FSK Modulation.

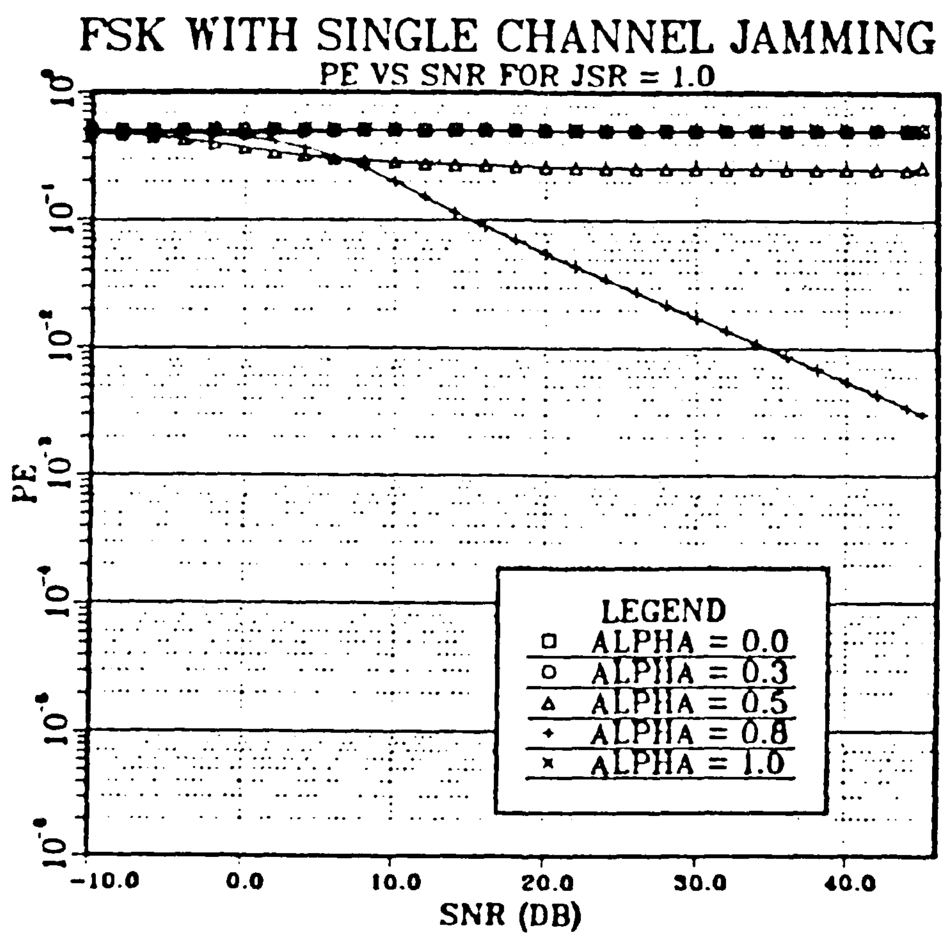


Figure 6.11 Variable Threshold Effect on Single Channel Jamming for FSK.

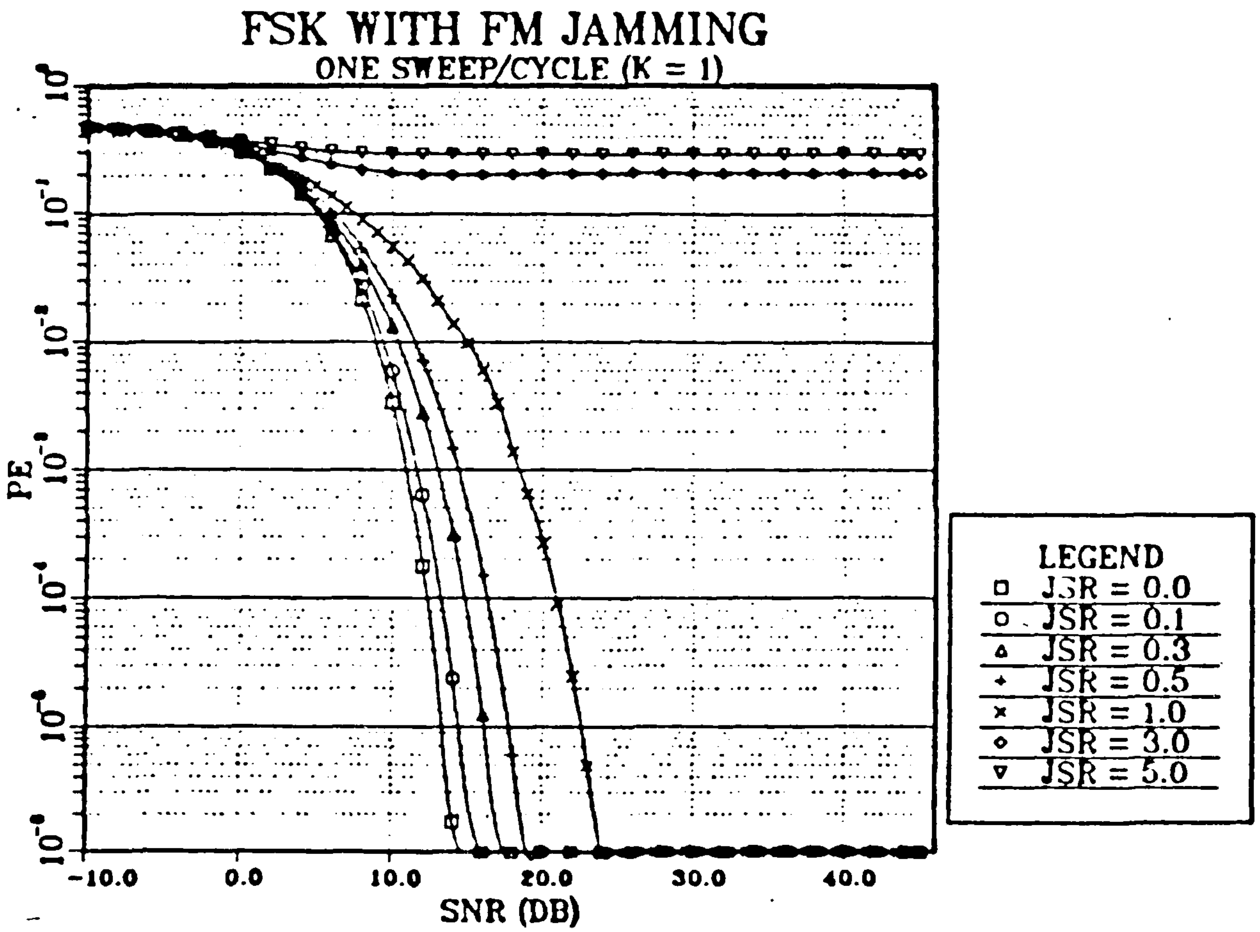


Figure 6.12 Frequency Modulated Jammer for FSK Modulation (K = 1).

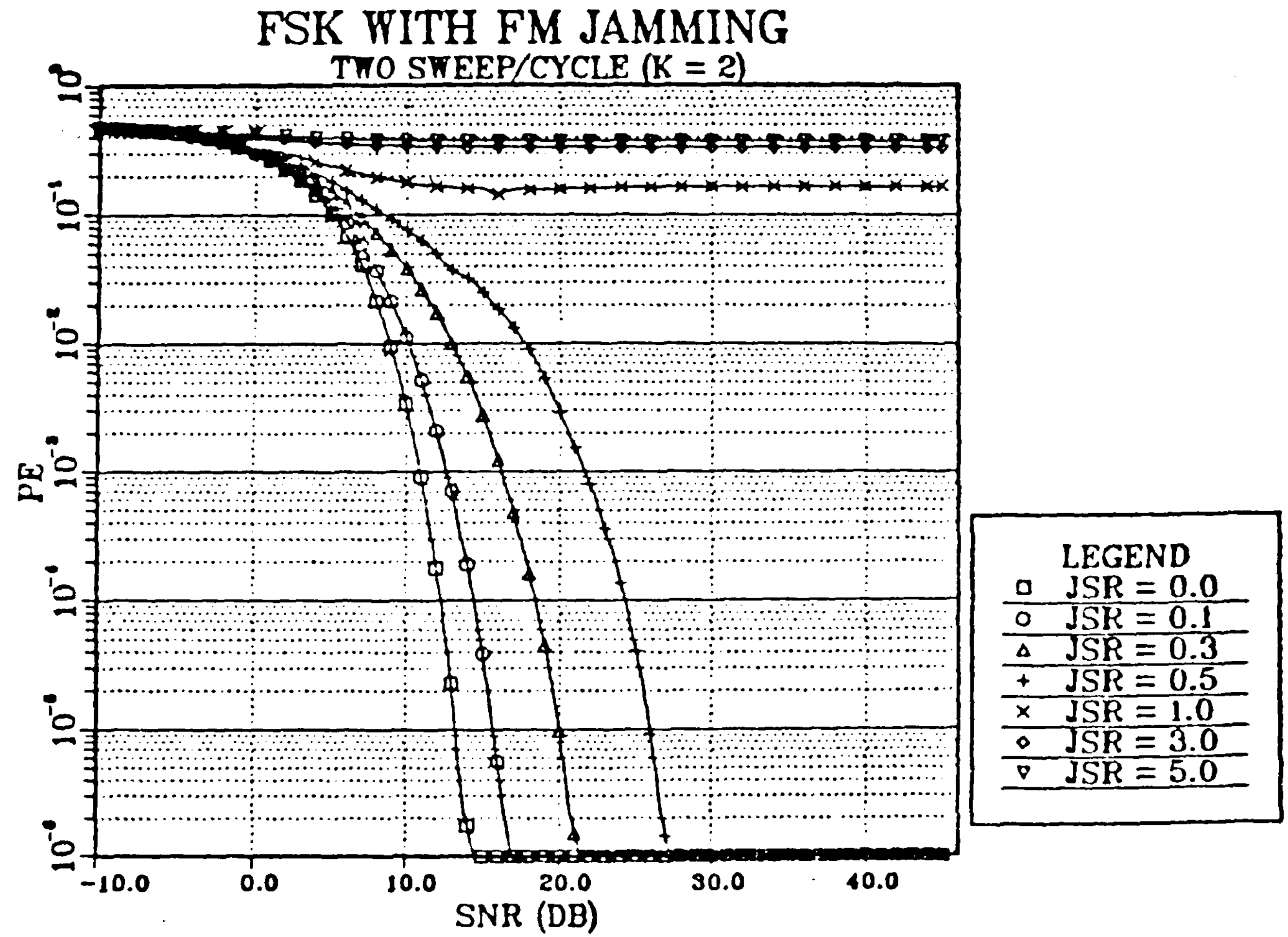


Figure 6.13 Frequency Modulated Jammer for FSK Modulation (K = 2).

APPENDIX A

DIGITAL COMPUTER IMPLEMENTATION OF THE MARCUM Q- FUNCTION

The Marcum Q-function occurs frequently in communication problems involving incoherent detection of signals with single or multiple observations in the presence of noise and jamming. So it is often necessary to compute values for the Marcum Q-function which is defined by

$$Q(\alpha, \beta) = \int_{\beta}^{\infty} V \exp\left(-\frac{V^2+\alpha^2}{2}\right) I_0(\alpha V)\, dV \tag{A.1}$$

$$= 1 - \int_{0}^{\beta} V \exp\left(-\frac{V^2+\alpha^2}{2}\right) I_0(\alpha V)\, dV$$

where $I_0(\cdot)$ is the modified Bessel function of zero order. It is noted that the integrand $f(V,\alpha)$ of Equation A.1 is the Rician density function which is sometimes called the generalized Rayleigh density function. The normalized Rician distribution for $\sigma^2 = 1$ is shown in Figure A.1 . For large values of α, the Rician density function f(V) can be approximated by the normalized Gaussian density function with mean value of approximately α. This can be justified as follows

$$f(V) = V \exp\left(-\frac{V^2+\alpha^2}{2}\right) I_0(\alpha V)\, dV$$

$$\cong V \exp\left(-\frac{V^2+\alpha^2}{2}\right) \cdot \frac{e^{\alpha V}}{\sqrt{2\pi\alpha V}}$$

$$= \frac{V}{\sqrt{2\alpha\pi V}} \exp\left[-\frac{(V-\alpha)^2}{2}\right] = \frac{1}{\sqrt{2\pi}} e^{-\frac{(v-\alpha)^2}{2}}$$

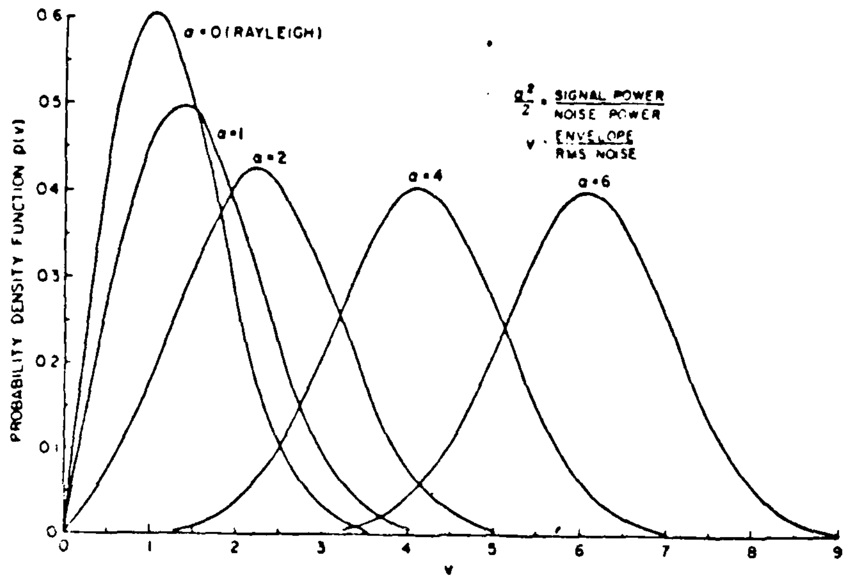


Figure A.1 Rician Density Function.

by use of the approximations $I_0(X) = \frac{e^{x}}{\sqrt{2\pi x}}$, and $\alpha V = V^2$ for large α.

In order to implement the computations of the Marcum Q-function using a digital computer, particular interest must be focused on values of the function on the tails of the Rician density defined by the integrand in Equation A.1 . Specifically, small values around the tails of the density must be monitored because of the limitations of the digital computer. By a change of variables $(X = \alpha V)$ the Marcum Q-function defined above can be expressed as

$$Q(\alpha, \beta) = 1 - \frac{1}{\alpha^2}\int_0^{\alpha\beta} x \exp\left[-\frac{(\frac{x}{\alpha} - \alpha)^2}{2}\right] f(x)\, dx \qquad (A.2)$$

where $f(X) = e^{-x} I_0(X)$. Since the digital computer can treat the values in the range of y such that $-180.218 \le y \le 174.643$ for the function e^{y} (for the IBM 3033), for digital computer implementation of the Marcum Q-function the

integral limits need to be adjusted to meet the conditions for these acceptable values in the computer. Negligible areas around the tails of the density beyond certain bounds must be discarded without significantly increasing overall error. Therefore from Equation A.2, the integral limits can be substituted with appropriate bounds such that exponent satisfies the condition given by

$$-\frac{(\frac{x}{\alpha}-\alpha)^2}{2} \geq -180$$

In other words, the variable X must be located within the range $\alpha^2 - \alpha\sqrt{360} \leq X \leq \alpha^2 + \alpha\sqrt{360}$. For a value of X beyond that range the exponential of Equation A.2 is too small (less than e^{-180}) compared to other computed values so that the area outside of the new limits can be neglected.

Let us denote UL and LL the upper and lower limit respectively defined by

$$UL = \alpha^2 + \alpha\sqrt{360} \quad ; \quad LL = \alpha^2 - \alpha\sqrt{360}$$

Then we can consider the various situations case by case. If the lower limit LL is positive for certain values of α, then in the case of $\alpha\beta < LL$, that is, $\beta < \alpha - \sqrt{360}$, the integral value can be ignored so that the value of $Q(\alpha, \beta)$ can be assigned to be one. In the case of $LL \leq \alpha\beta \leq UL$, that is, $\alpha - \sqrt{360} \leq \beta \leq \alpha + \sqrt{360}$, the Marcum Q-function can be approximated by

$$Q(\alpha, \beta) = 1 - \frac{1}{\alpha^2}\int_{LL}^{\alpha\beta} x \exp\left[-\frac{(\frac{x}{\alpha}-\alpha)^2}{2}\right] f(x)\, dx$$

In the case of $\alpha\beta > UL$, that is, $\beta > \alpha + \sqrt{360}$, the Marcum Q-function can be computed from

$$Q(\alpha, \beta) = 1 - \frac{1}{\alpha^2}\int_{LL}^{UL} x \exp\left[-\frac{(\frac{x}{\alpha}-\alpha)^2}{2}\right] f(x)\, dx$$

On the other hand if the lower limit LL is negative for some value of α, then there are two cases to be considered. In the case of $\alpha\beta \le$ UL, that is, $\beta \le \alpha + \sqrt{360}$, the Marcum Q-function can be substituted with the value given by

$$Q(\alpha, \beta) = 1 - \frac{1}{\alpha^2}\int_0^{\alpha\beta} x \exp\left[- \frac{(\frac{x}{\alpha} - \alpha)^2}{2} \right] f(x)\, dx$$

Finally in the case of $\alpha\beta \ge$ UL, that is, $\beta \ge \alpha + \sqrt{360}$, the value of Marcum Q-function can be computed from

$$Q(\alpha, \beta) = 1 - \frac{1}{\alpha^2}\int_0^{UL} x \exp\left[- \frac{(\frac{x}{\alpha} - \alpha)^2}{2} \right] f(x)\, dx$$

Here it has been assumed that the function f(X) defined by $e^{-X} I_0(X)$ does not impose the limitation of computation on the digital computer and that the library functions for the computation of $e^{-X} I_0(X)$ and its integration with desired accuracy are available to the user.

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